CDCC analysis for momentum distribution of ^{22}Mg in ^{23}Al+^{12}C reaction

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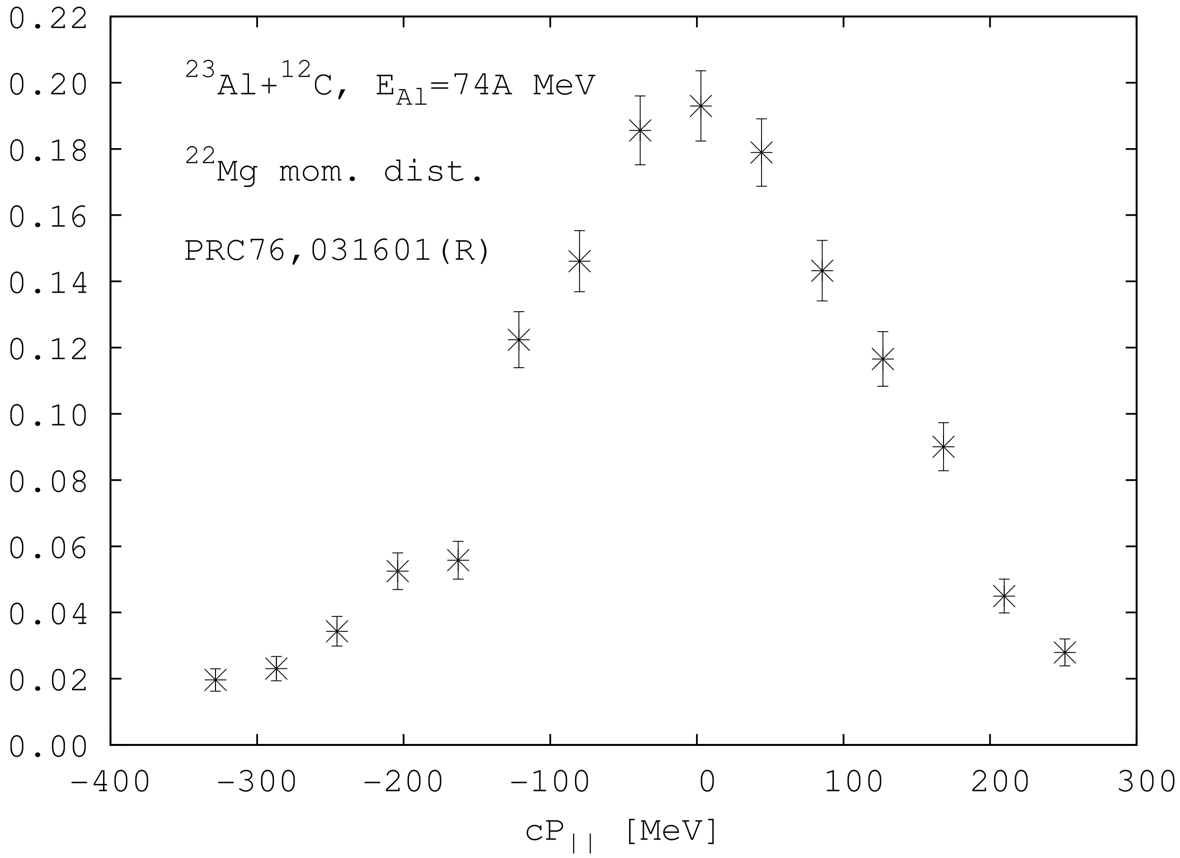
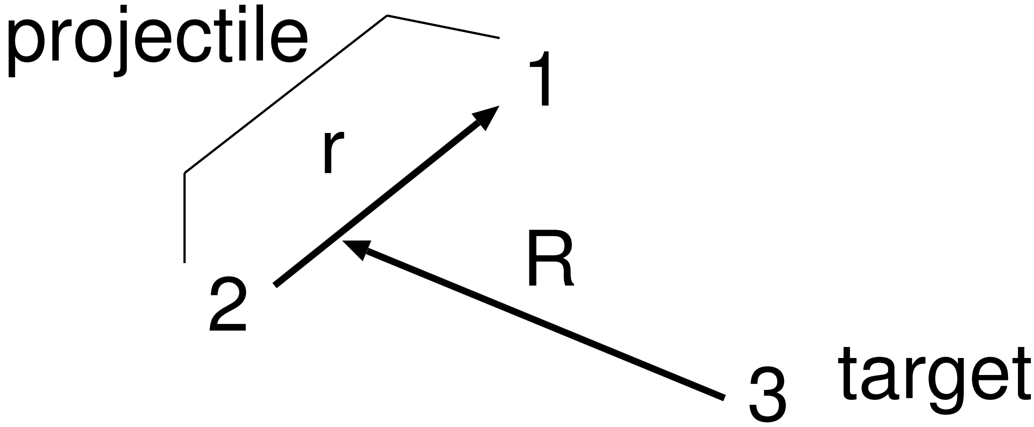


lets write a CDCC program and analyze the data

CDCC is a 3 body like approach to scatt. of loosely bound projectile off the hard nucleus



int.	pot. type	states
V_{12}	real pot.	bound/ scatt.
V_{13}	OP pot.	scatt.
V_{23}	OP pot.	scatt.

Hamiltonian H:

$$H = T_r + V_{12}(r)$$

$$+ T_R + V_{13}(r_{13}) + V_{23}(r_{23})$$

wave function $\phi(r)$ be defined by

$$\{T_r + V_{12}(\mathrm{r}) - E_c\}\,\phi_c(\mathrm{r}) = 0$$

scatt. state wf. at large r

$$\phi_c(k,\, r) \longrightarrow \sqrt{\frac{2}{\pi}}\,\sin(k\,r - l_c\pi/2 + \delta_{c\,k})/r$$

k	wave number
l_c	orb. ang. mom. for ch. c
$\delta_{c\,k}$	nucl. phase shift

if charge and spin are neglected

truncation	k and spin space
discretization	k space

hence

Continuum Discretized
Coupled Channels (CDCC)

$$\hat{\phi}_{c\,j} = \frac{1}{\sqrt{k_j - k_{j-1}}} \int_{k_{j-1}}^{k_j} \phi_c(k,\, r)\, dk$$

and is orthonormalized as

$$\langle \hat{\phi}_{c\,j} \,|\, \hat{\phi}_{c'\,j'} \rangle_{\rm r} = \delta_{c\,c'} \delta_{j\,j'}$$

suffix	meaning
c, c’	spins
j, j’	wave numbers

$V_{13} + V_{23}$:
consists of nucl. and Coul. pots.

they are expanded by
Legendre functions P_λ

$$V_{13}(r_{13}) + V_{23}(r_{23}) = \sum_{\lambda} v_\lambda(r,\, R)\, P_\lambda(\hat{\mathrm{r}} \cdot \hat{\mathrm{R}})$$

λ multipolarity

$\lambda = 0$ equivalent(folded) potential
pre/post acceleration

$\lambda \geq 1$ tidal force
dipole/multipole break up
reorientation

eigen state of H with angular momentum J, M

$$\Psi_{J\,M} = \frac{1}{R}\sum_{c,\,j}\ \chi^{J}_{L_{c\,j}}(R) \times [\hat{\phi}_{c\,j}(r)\ i^{l_c}\,Y_{l_c}(\hat{\mathrm{r}})\ i^{L_{c\,j}}\ Y_{L_{c\,j}}(\hat{\mathrm{R}})]_{J\,M}$$

$[\ \ldots\ \ ]$ for ang. mom. coupling

$\chi^{J}_{L_{c\,j}}(R)$: motion of (1-2) and 3, satisfies the CDCC eq.

$$(T_R - E_{c\,j})\chi^{J}_{L_{c\,j}}(R) = -\sum_{c'\,j'}\langle[c\,j]_J|(V_{13}+V_{23})|[c'\,j']_J\rangle \times \chi^{J}_{L'_{c'\,j'}}(R)$$

CDCC eq. is solved numerically with usual boundary cond.

$$\chi^{J}_{L_{cj}}(R) \to I_{L_{c_0}}\, \delta_{c,c_0}\, \delta_{L_{cj},L_{c_0}} - \sqrt{\frac{K_{c_0}}{K_{cj}}}\, S^{J}_{cj,\, c_0 L_{c_0}}\, O_{cj}$$

where c_0 stands for inc. ch.

I_L (O_L) are usual Coul. wf.

Elastic cross sec

$$\boldsymbol{\sigma_{el} \propto |f_c + \sum (\text{geom. factor})\, e^{i(\sigma_{L_0}+\sigma_L)} \times \left(S^J_{L,L_0} - \delta_{L\,L_0}\right) Y_{LM}(\hat{\mathrm{R}})|^2}$$

triple diff. cross sec.

$$\boldsymbol{\frac{d^3\sigma}{d\Omega_1\, d\Omega_2\, dE_1} \propto (\text{final state dens.}) \times |T|^2}$$

$$\boldsymbol{T \propto \sum_{J\,M\,c} (\text{ geom. factor}) \left(\frac{e^{i(\delta_{c\,k}+\sigma_{c\,k})}}{k}\right) \times \left(e^{i(\sigma_{L_0}+\sigma_{L_c})}\, S^J_{L_c\,L_0}(k)\right) \times [\mathbf{Y}_{l_c}(\hat{\mathrm{r}}) \otimes \mathbf{Y}_{L_c}(\hat{\mathrm{R}})]_{J\,M}}$$

numerical aspects

^{23}Al+^{12}C at $E_{\rm Al}$= 80A MeV

assumed: $^{23}\mathrm{Al} = \mathrm{p} + {}^{22}\mathrm{Mg}$

pot. V_{12} between p and ^{22}Mg

central Woods-Saxon type

Coul. uniform. charged sphere

spin-orbit Thomas type

with common geometric param.

following energies are reproduced

level	spin	E [MeV]	width
gs	$d_{5/2}$	-0.125	
1st res.	$s_{1/2}$	0.402	77 [eV]
2nd res.	$d_{3/2}$	2.444	76 [keV]

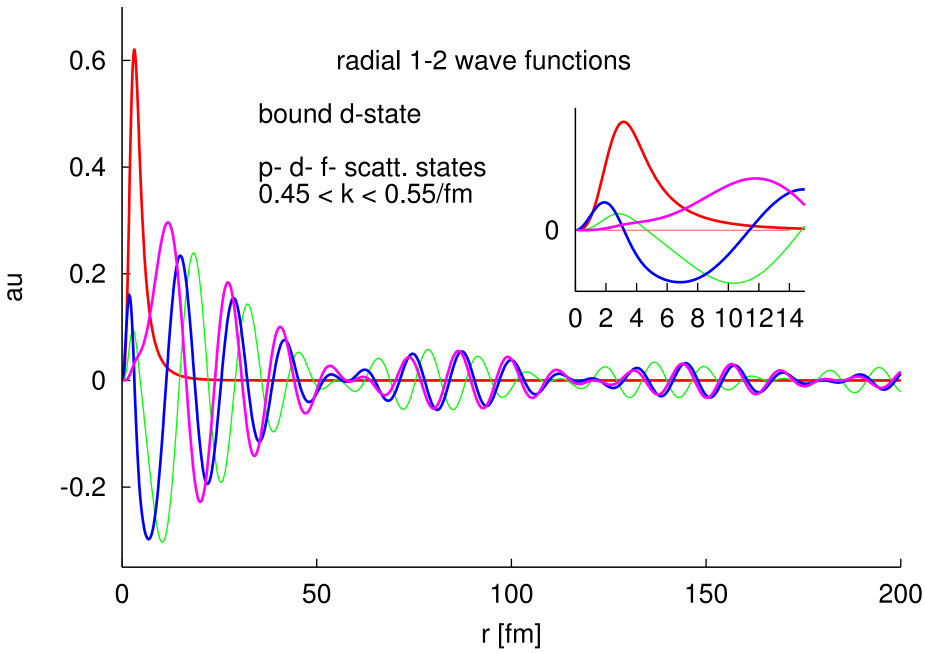
intrinsic spins are neglected

to reduce cpu time

$\hat{\phi}(r)$, the 1-2 base

real, even for scatt. states

***d*-, *p*-, *f*-state (bound/ scatt.) wf.**
$0.45 < k < 0.55\ fm^{-1}$

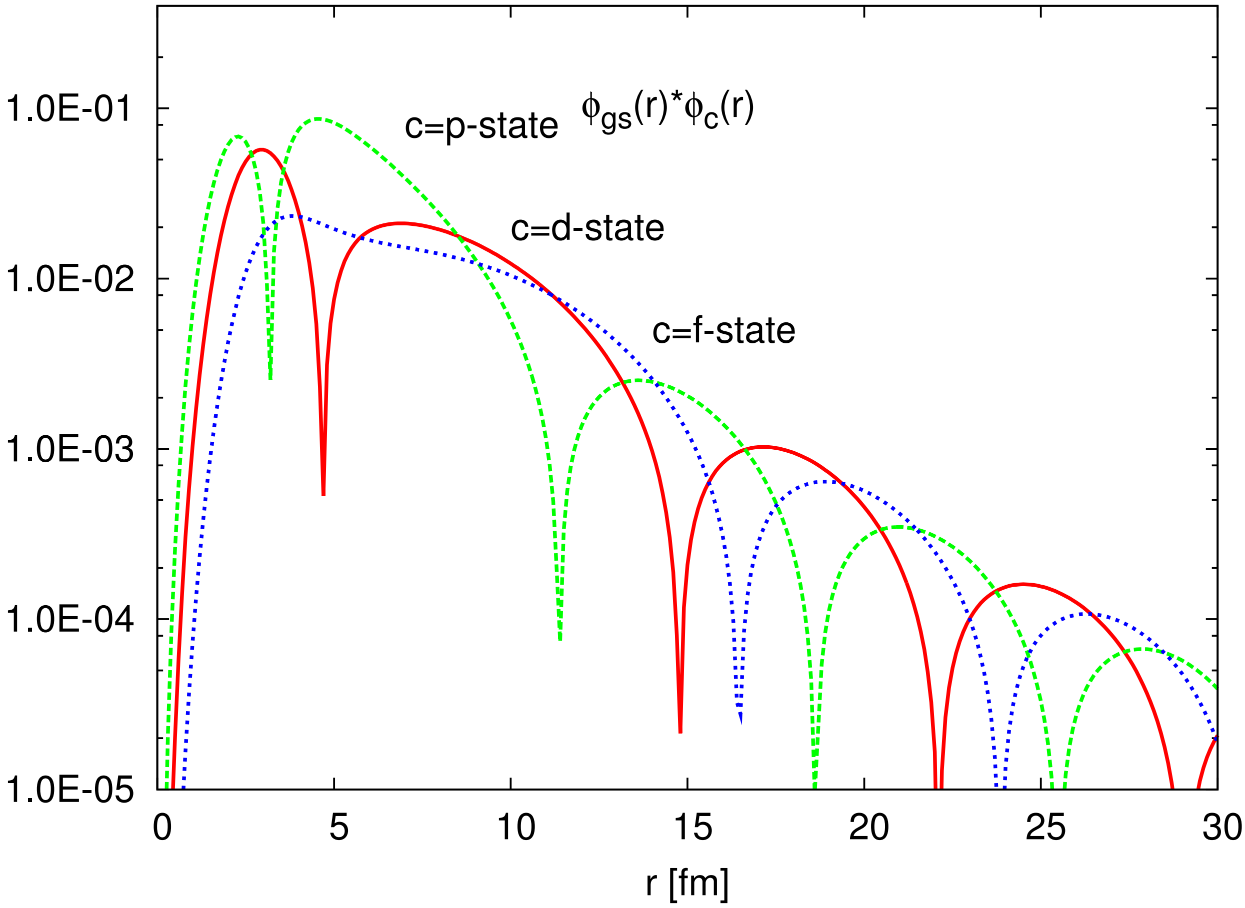


dumps at large r, which is a large merit of binning

s, p, d-states have a node $r < 4$ fm but not for f-state(No bond states)

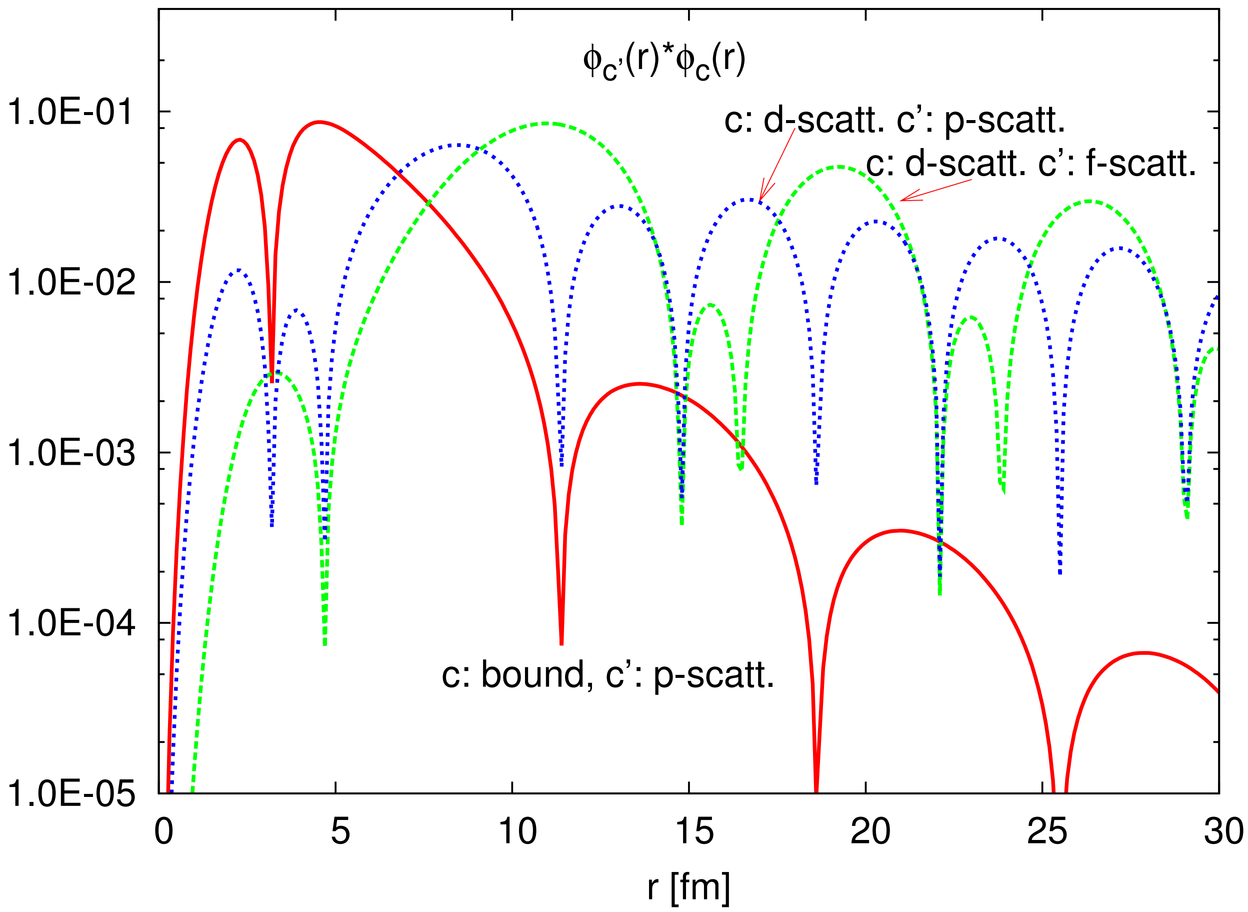
$\underline{\text{product of } \hat{\phi}}$

(bound) × (scatt.) state wf.



this product is related to break up reaction

prod. of continuum $\hat{\phi}$'s



break up is induced at small r
but at large r
cont.-cont. coupling is important
and is called "post acceleration"

V_{13} and V_{23} :

OP pot. are used

$V_{(\mathrm{p}-{}^{12}C)}$	**Watson et al.,**
$V_{({}^{22}\mathrm{Mg}-{}^{12}\mathrm{C})}$	**Beunerd et al.,**
$\Uparrow$	**remains ambiguity**

if we require $\hat{\phi}$ feels nucleus

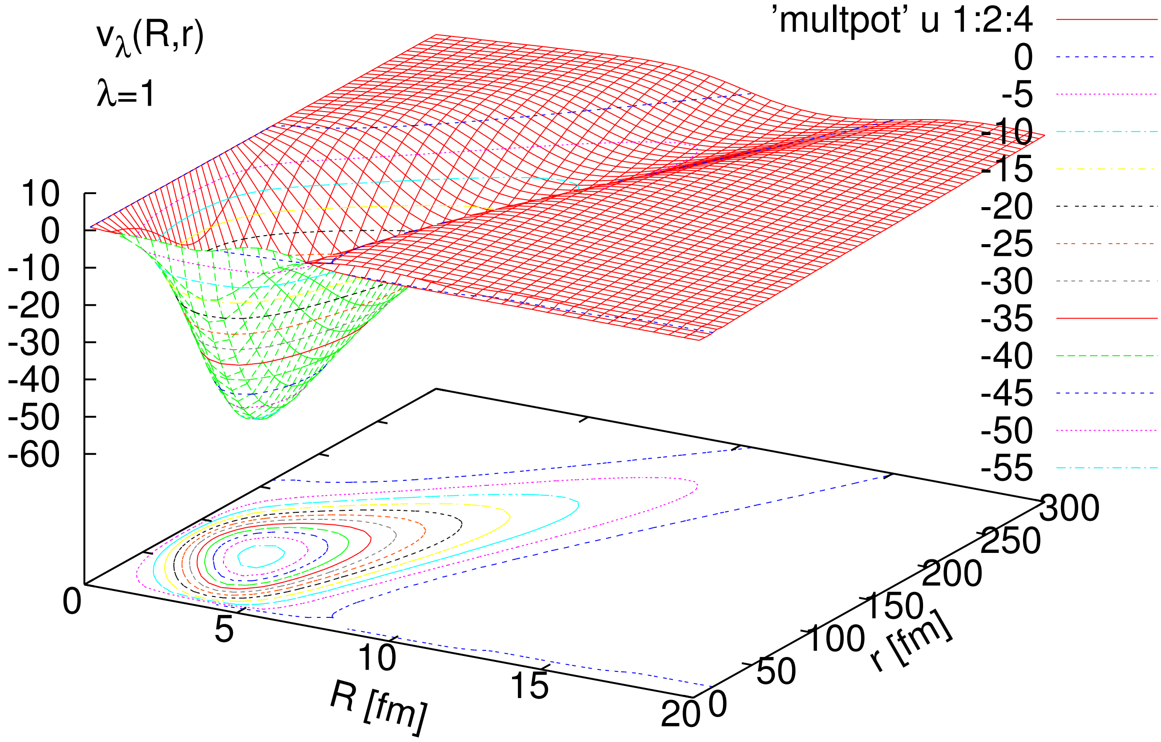
$$r_{max} \geq \frac{m_{\mathrm{Mg}} + m_{\mathrm{p}}}{m_{\mathrm{p}}} R_{max} \; !$$

No excitation of ^{12}C nor ^{22}Mg

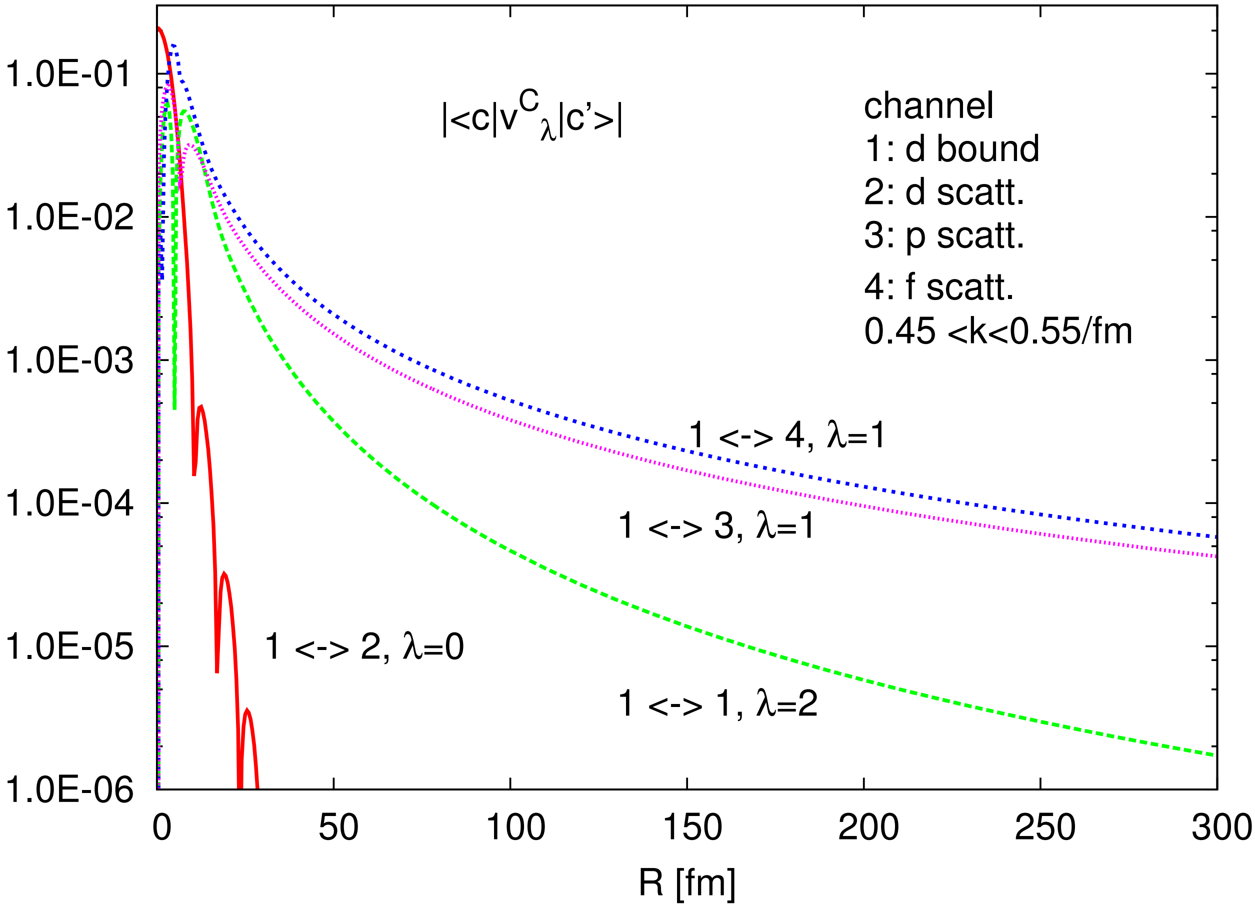
No coalescence

No Coul. BU included in the final analysis !

dipole part of potential



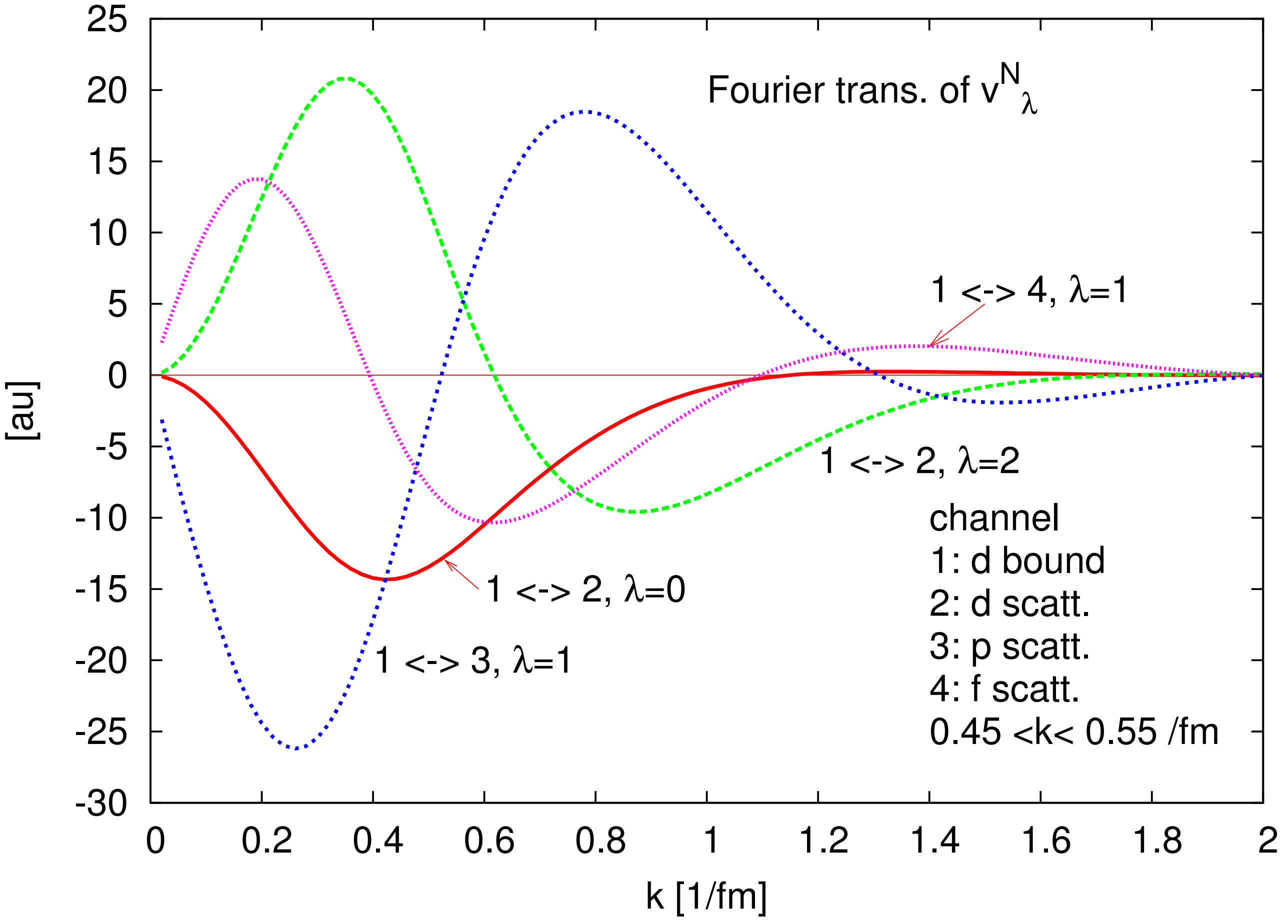
$$\langle \hat{\phi}_c(r) | v_\lambda^C(r, R) | \hat{\phi}_{c'}(r) \rangle_r$$



off diag. monopole elem. dumps
very rapidly,
due to orthogonality

$\lambda(>0)$ elements dump as
$R^{-(\lambda+1)}$ for large r

Fourier exp. of $v_\lambda^N(r,\ R)$



$\lambda = 0,\ 1$ and 2 component for

$c <-> c'$ transition

*** CDCC eq. solved numerically**
by using 8 point Stömer's method

*** safeguard**
integrate NOT from the origin

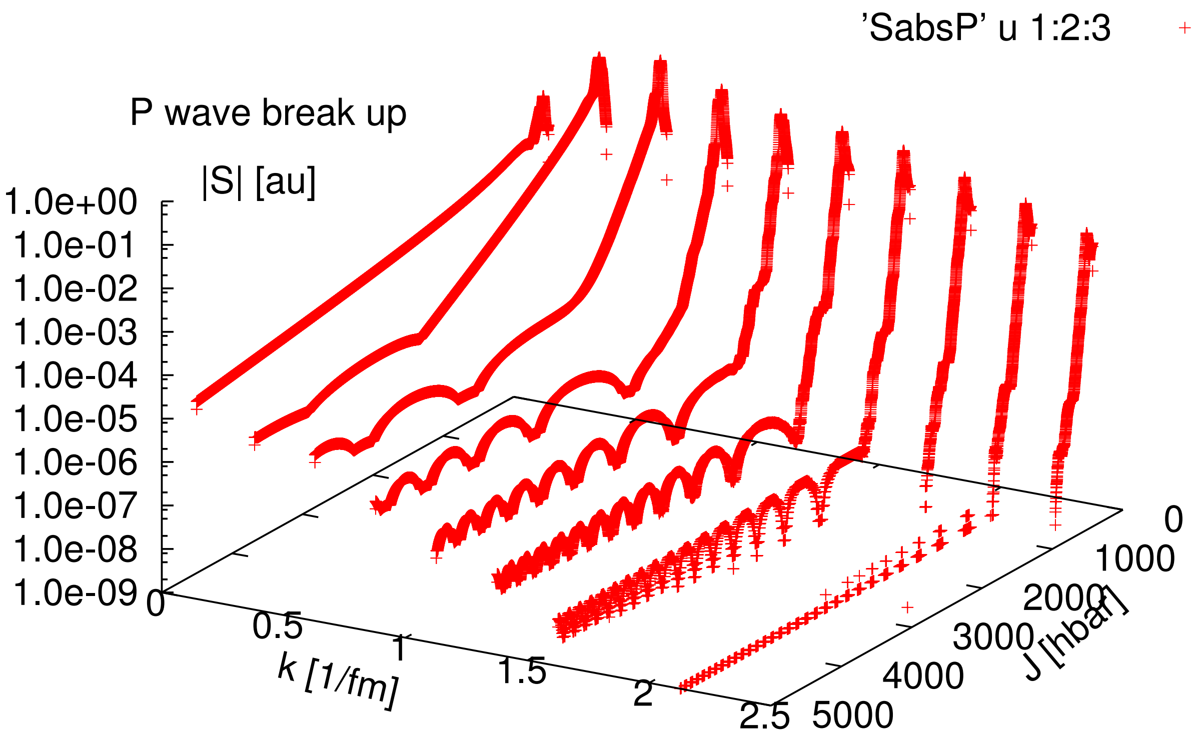
*** to get indep. sol. vectors**
occasional ortho-normalization

*** S-mat.: cond. num. estimated**

*** Coul. wf. Virmigham approach.**
use of continued fraction

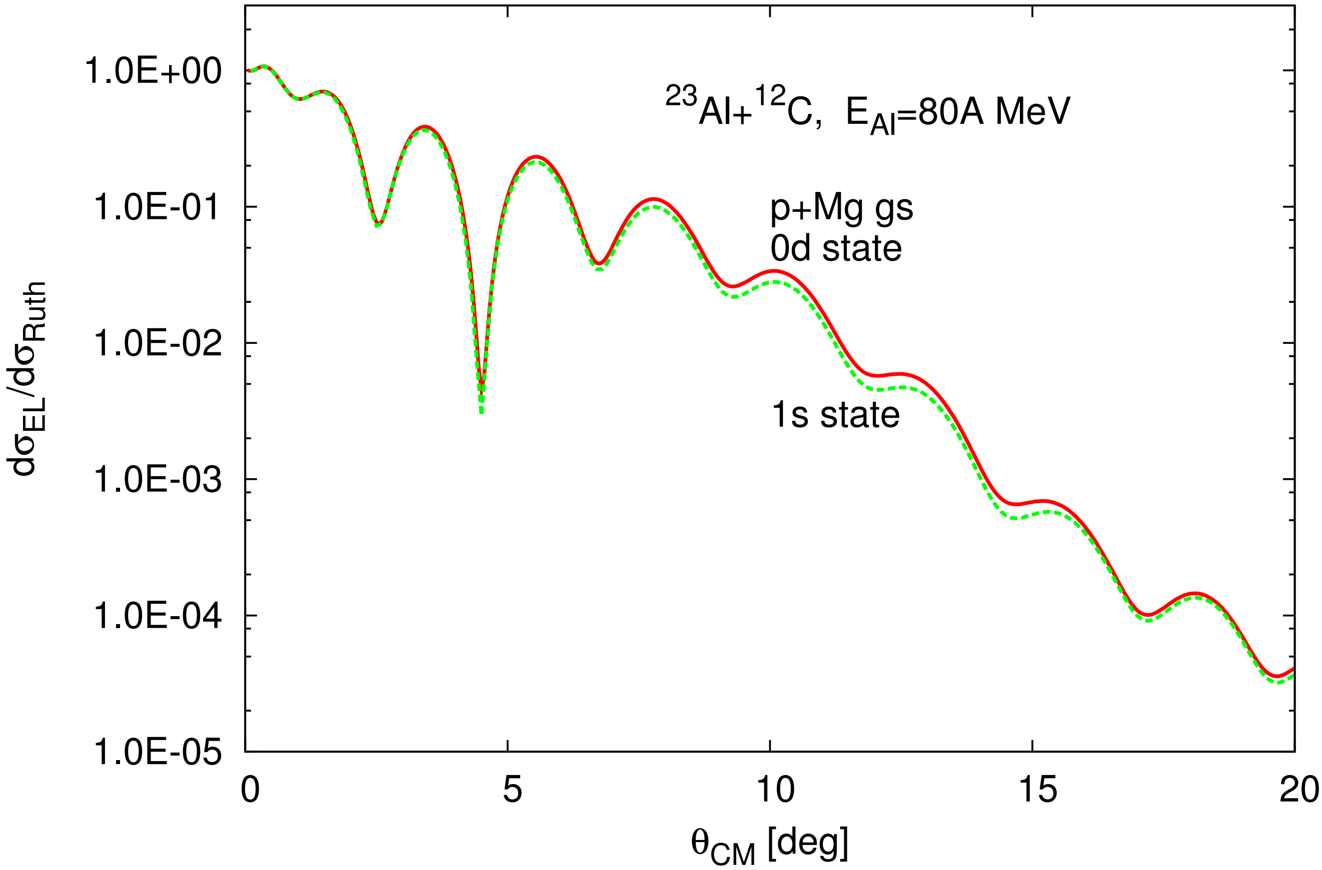
*** 3- 6-j: use of 3 term recur. relation**
purge factorial evaluation

J and k dep. of *p*-wave absorption



just qualitative !

elastic scatt. cross sec.



l=0 to 4 states of 1-2 system

$0 < k < 1.5$ fm^{-1}

No Coul. break up

gs. of ^{23}Al $\left\{ \begin{array}{l} \pi 0d \text{ state} \\ \pi 1s \text{ state} \end{array} \right.$

No exp. data

triple diff. cross sec.

No Coul. BU inluded yet

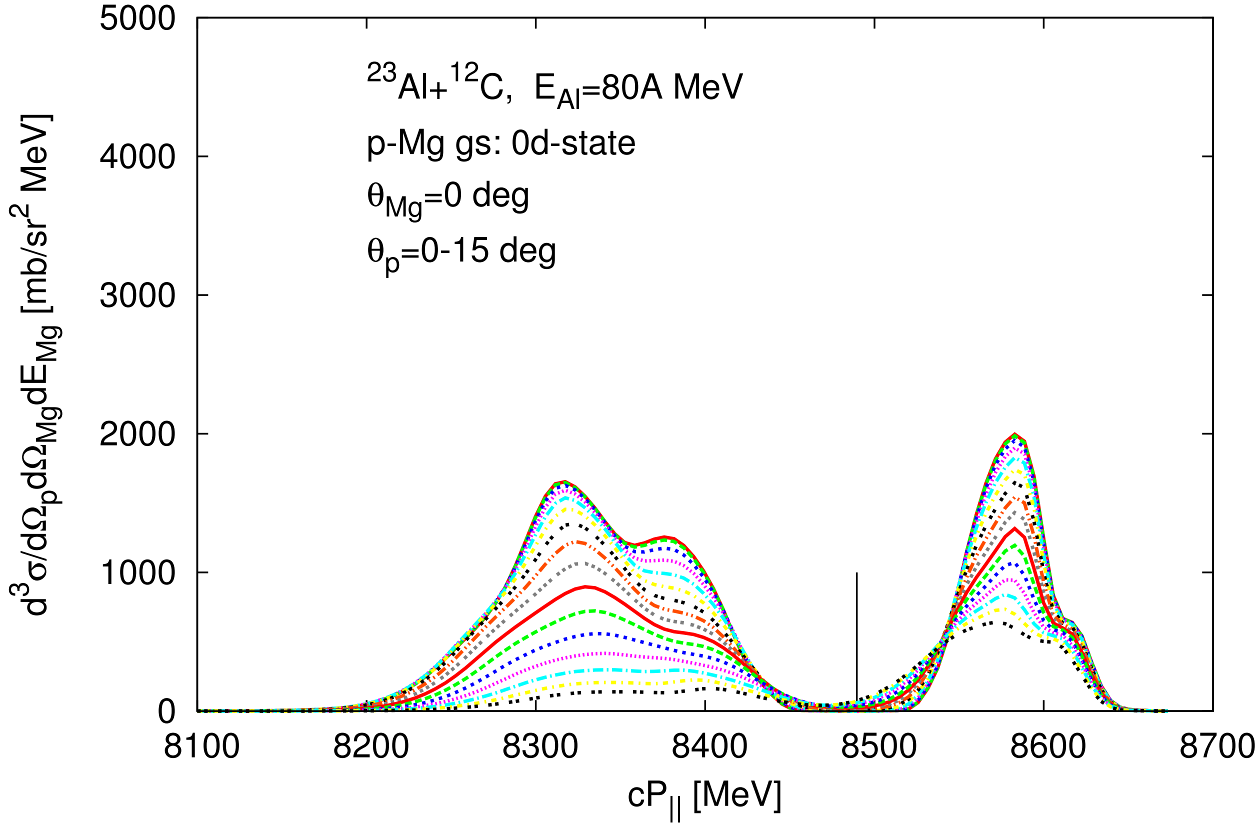
s-, d-state for p-^{22}Mg bound state

NOT symmetric about $k = 0$

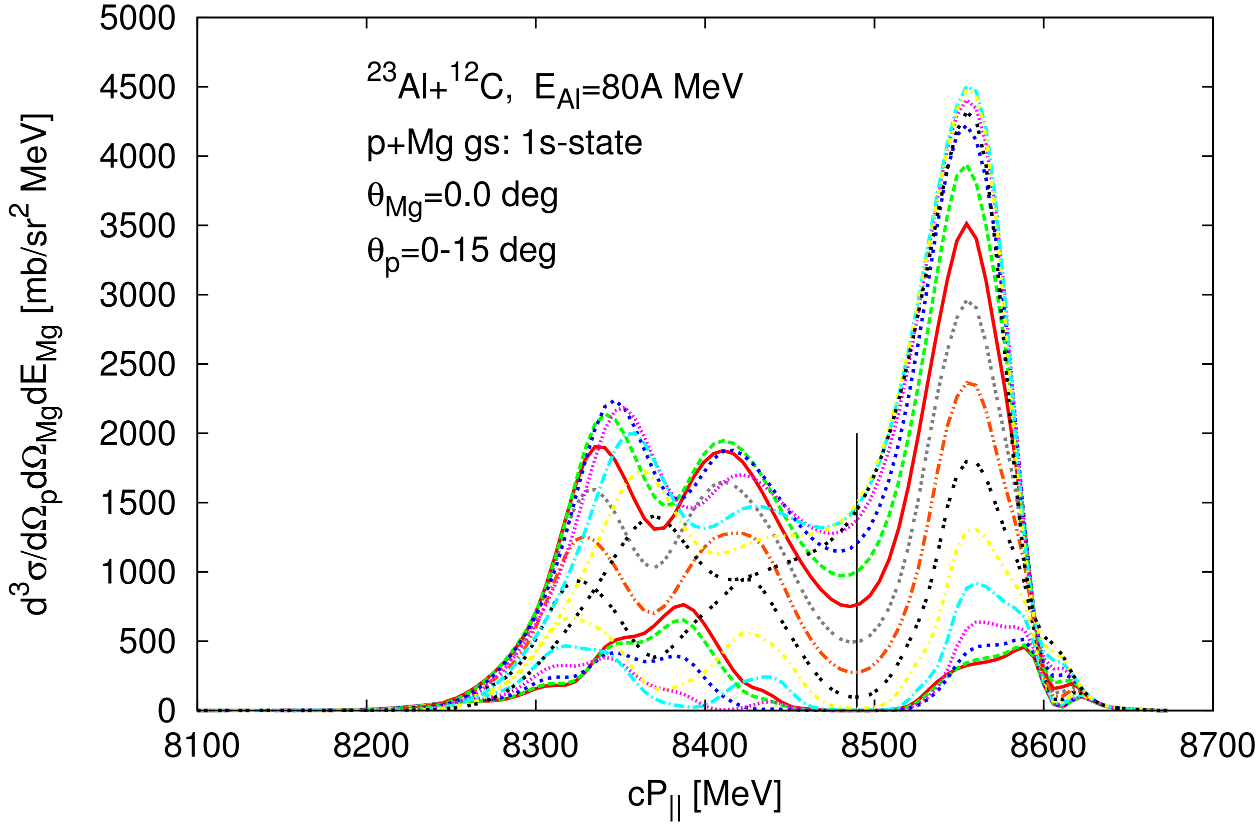
central supression

- **dipole break up dominates**
- **Coulomb supression of $\hat{\phi}$**
- **Coul. BU may fill the dip**

d-state be preferred for p+^{22}Mg gs



^{23}Al+^{12}C, E$_{Al}$=80A MeV
p-Mg gs: 0d-state
θ_{Mg}=0 deg
θ_p=0-15 deg
$d^3\sigma/d\Omega_p d\Omega_{Mg} dE_{Mg}$ [mb/sr^2 MeV]
cP$_{||}$ [MeV]



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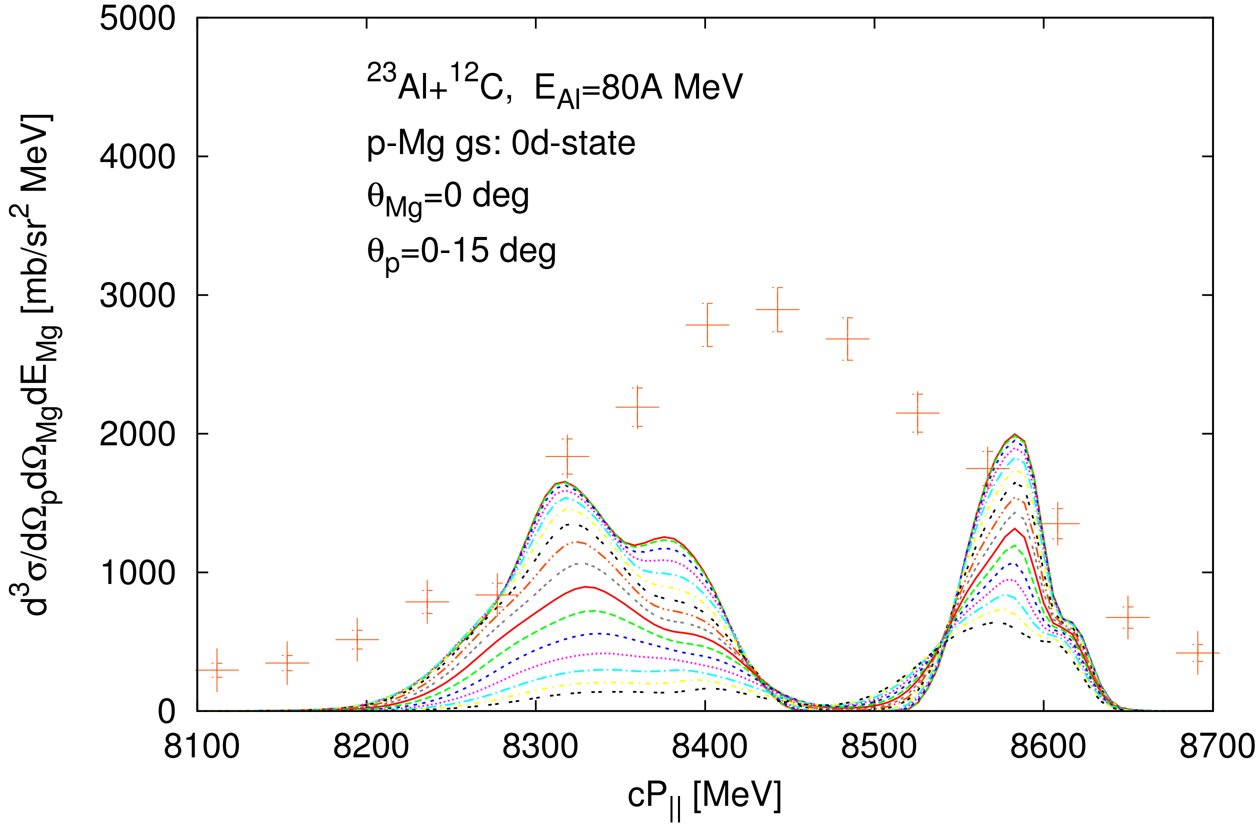
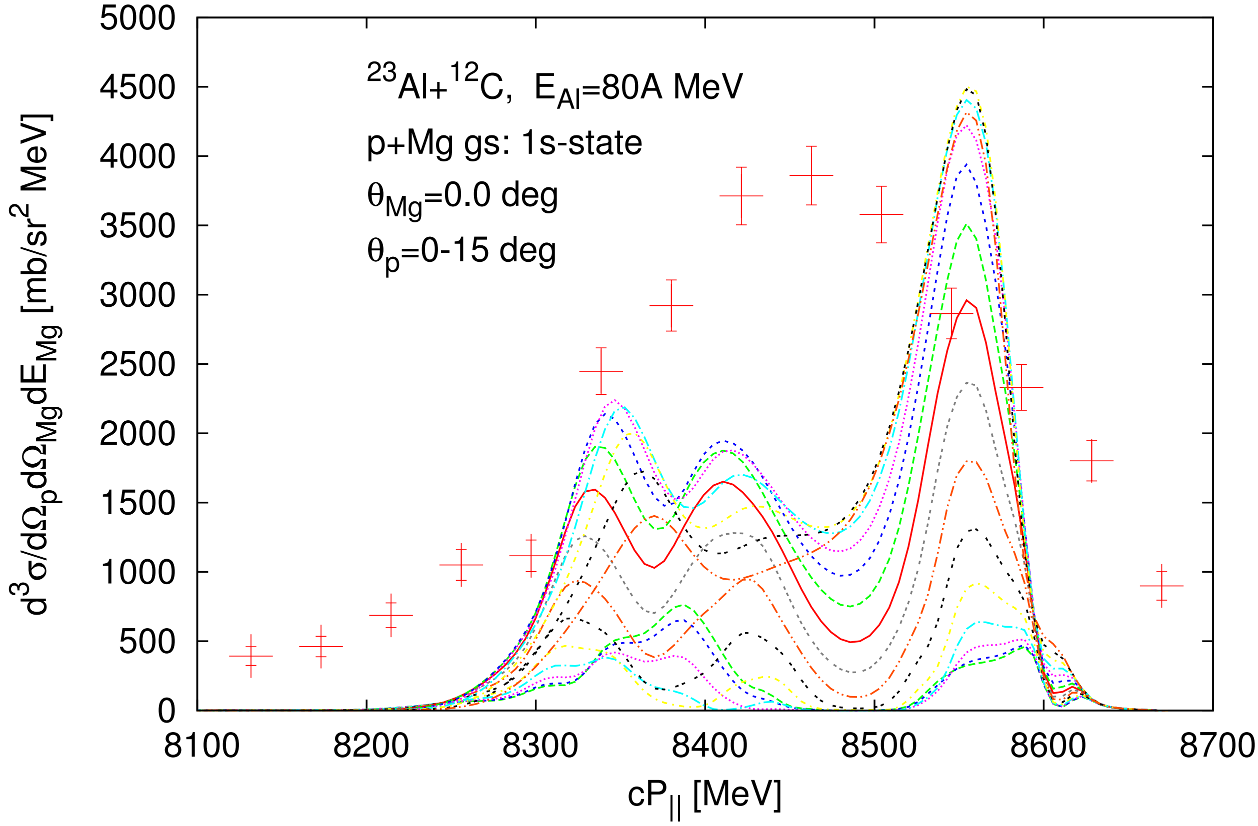


^{23}Al+^{12}C, E_{Al}=80A MeV
p-Mg gs: 0d-state
θ_{Mg}=0 deg
θ_p=0-15 deg
$d^3\sigma/d\Omega_p d\Omega_{Mg} dE_{Mg}$ [mb/sr^2 MeV]
cP$_{||}$ [MeV]
0
1000
2000
3000
4000
5000
8100
8200
8300
8400
8500
8600
8700



^{23}Al+^{12}C, E_{Al}=80A MeV
p+Mg gs: 1s-state
θ_{Mg}=0.0 deg
θ_p=0-15 deg
$d^3\sigma/d\Omega_p d\Omega_{Mg} dE_{Mg}$ [mb/sr^2 MeV]
cP$_{||}$ [MeV]
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
8100
8200
8300
8400
8500
8600
8700