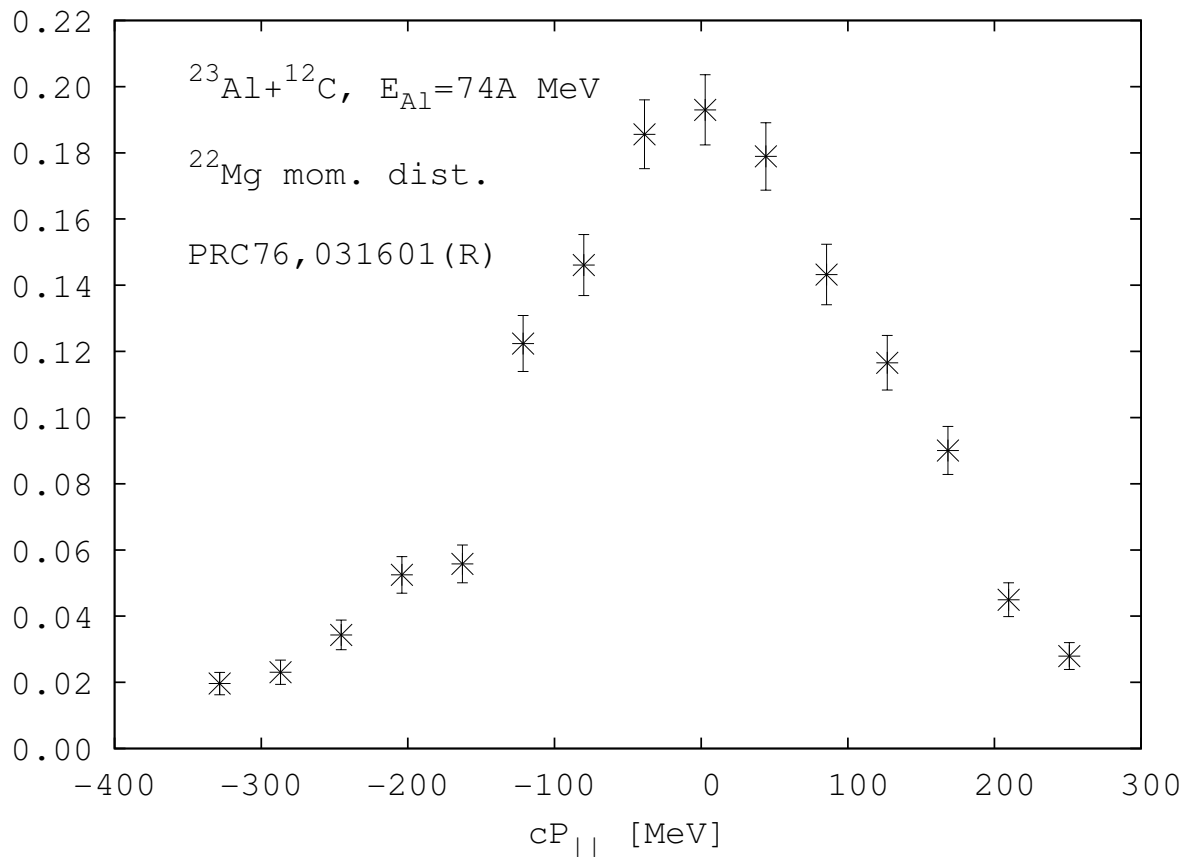


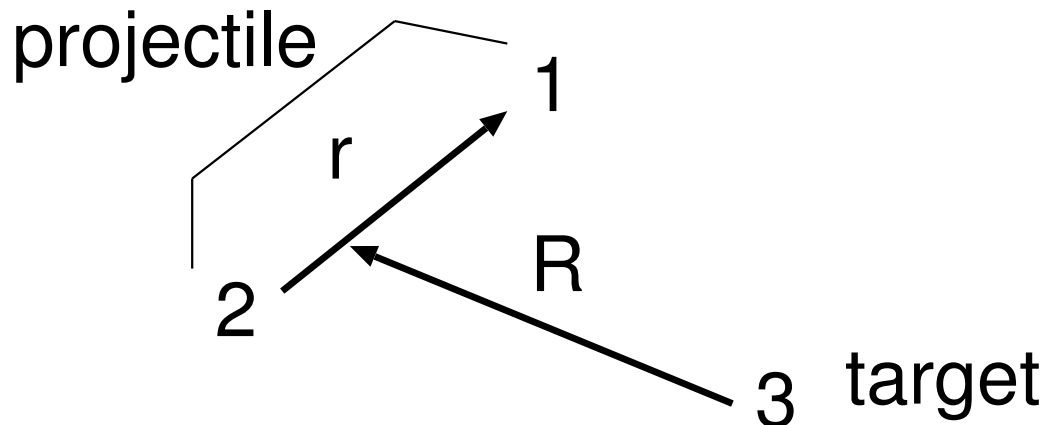
CDCC analysis for momentum distribution of ^{22}Mg in $^{23}\text{Al}+^{12}\text{C}$ reaction

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lets write a CDCC program and
analyze the data

CDCC is a 3 body like approach to
scatt. of loosely bound projectile
off the hard nucleus



int.	pot. type	states
V_{12}	real pot.	bound/ scatt.
V_{13}	OP pot.	scatt.
V_{23}	OP pot.	scatt.

Hamiltonian H :

$$\begin{aligned}
 H = & T_r + V_{12}(r) \\
 & + T_R + V_{13}(r_{13}) + V_{23}(r_{23})
 \end{aligned}$$

wave function $\phi(r)$ be defined by

$$\{T_r + V_{12}(r) - E_c\} \phi_c(r) = 0$$

scatt. state wf. at large r

$$\phi_c(k, r) \rightarrow \sqrt{\frac{2}{\pi}} \sin(kr - l_c\pi/2 + \delta_{ck})/r$$

k wave number

l_c orb. ang. mom. for ch. c

δ_{ck} nucl. phase shift

if charge and spin are neglected

truncation	k and spin space
discretization	k space

hence

Continuum Discretized
Coupled Channels (CDCC)

$$\hat{\phi}_{c j} = \frac{1}{\sqrt{k_j - k_{j-1}}} \int_{k_{j-1}}^{k_j} \phi_c(k, r) dk$$

and is orthonormalized as

$$\langle \hat{\phi}_{c j} | \hat{\phi}_{c' j'} \rangle_r = \delta_{c c'} \delta_{j j'}$$

suffix	meaning
c, c'	spins
j, j'	wave numbers

$V_{13} + V_{23}$:
consists of nucl. and Coul. pots.

they are expanded by
Legendre functions P_λ

$$\begin{aligned} V_{13}(r_{13}) + V_{23}(r_{23}) \\ = \sum_{\lambda} v_{\lambda}(r, R) P_{\lambda}(\hat{\mathbf{r}} \cdot \hat{\mathbf{R}}) \end{aligned}$$

λ multipolarity

$\lambda = 0$ equivalent(folded) potential
pre/post acceleration

$\lambda \geq 1$ tidal force
dipole/multipole break up
reorientation

eigen state of H with
angular momentum J, M

$$\Psi_{J M} = \frac{1}{R} \sum_{c, j} \chi_{L_{c j}}^J(R) \\ \times [\hat{\phi}_{c j}(r) i^{l_c} Y_{l_c}(\hat{r}) i^{L_{c j}} Y_{L_{c j}}(\hat{R})]_{J M}$$

[...] for ang. mom. coupling

$\chi_{L_{c j}}^J(R)$: motion of (1-2) and 3,
satisfies the CDCC eq.

$$(T_R - E_{c j}) \chi_{L_{c j}}^J(R) \\ = - \sum_{c' j'} \langle [c j]_J | (V_{13} + V_{23}) | [c' j']_J \rangle \\ \times \chi_{L'_{c' j'}}^J(R)$$

CDCC eq. is solved numerically
with usual boundary cond.

$$\chi_{L_{c_j}}^J(R) \rightarrow I_{L_{c_0}} \delta_{c,c_0} \delta_{L_{c_j},L_{c_0}} \\ - \sqrt{\frac{K_{c_0}}{K_{c_j}}} S_{c_j,c_0}^J L_{c_0} O_{c_j}$$

where c_0 stands for inc. ch.

I_L (O_L) are usual Coul. wf.

Elastic cross sec

$$\sigma_{el} \propto |f_c + \sum (\text{geom. factor}) e^{i(\sigma_{L_0} + \sigma_L)} \\ \times \left(S_{L, L_0}^J - \delta_{L L_0} \right) Y_{LM}(\hat{\mathbf{R}})|^2$$

triple diff. cross sec.

$$\frac{d^3 \sigma}{d\Omega_1 d\Omega_2 dE_1} \propto (\text{final state dens.}) \times |T|^2$$

$$T \propto \sum_{J M c} (\text{geom. factor}) \left(\frac{e^{i(\delta_{c k} + \sigma_{c k})}}{k} \right) \\ \times \left(e^{i(\sigma_{L_0} + \sigma_{L_c})} S_{L_c L_0}^J(k) \right) \\ \times [Y_{l_c}(\hat{\mathbf{r}}) \otimes Y_{L_c}(\hat{\mathbf{R}})]_{J M}$$

numerical aspects

$^{23}\text{Al} + ^{12}\text{C}$ at $E_{\text{Al}} = 80 A \text{ MeV}$

assumed: $^{23}\text{Al} = \text{p} + ^{22}\text{Mg}$

pot. V_{12} between p and ^{22}Mg

central Woods-Saxon type

Coul. uniform. charged sphere

spin-orbit Thomas type

with common geometric param.

following energies are reproduced

level	spin	E [MeV]	width
gs	$d_{5/2}$	-0.125	
1st res.	$s_{1/2}$	0.402	77 [eV]
2nd res.	$d_{3/2}$	2.444	76 [keV]

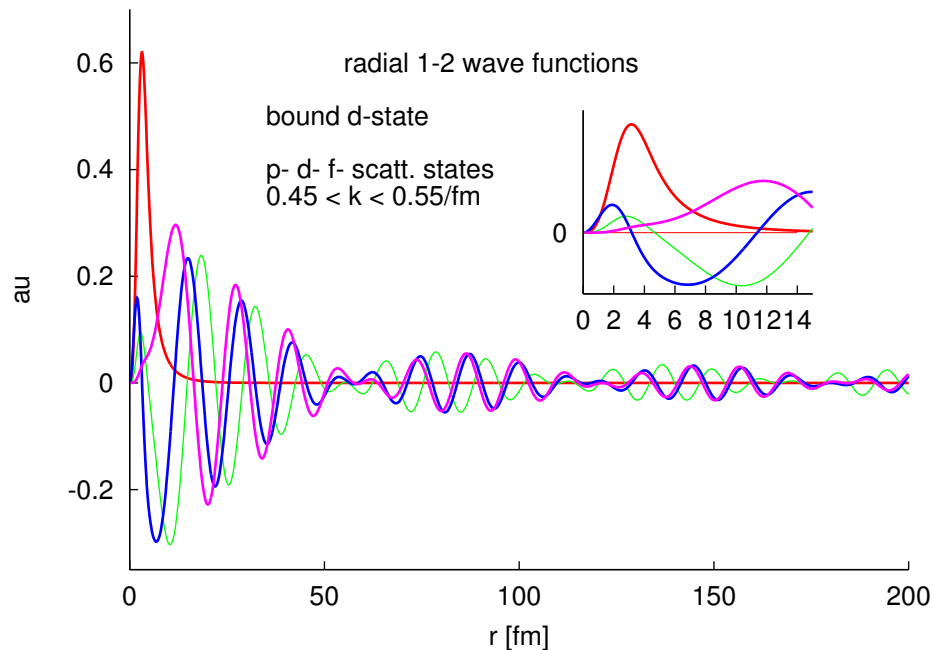
intrinsic spins are neglected

to reduce cpu time

$\hat{\phi}(r)$, the 1-2 base

real, even for scatt. states

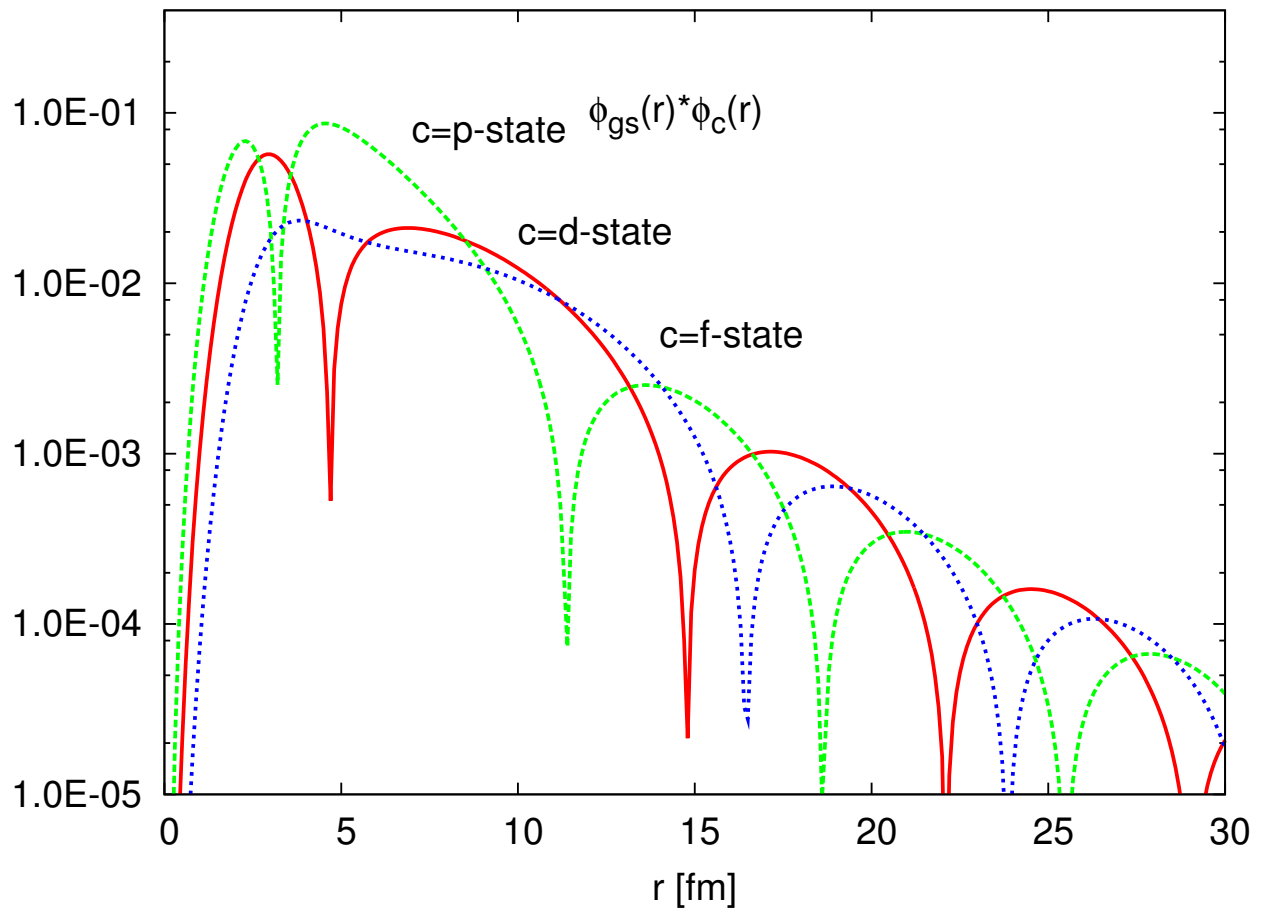
d -, p -, f -state (bound/ scatt.) wf.
 $0.45 < k < 0.55 \text{ fm}^{-1}$



dumps at large r , which is a
 large merit of binning

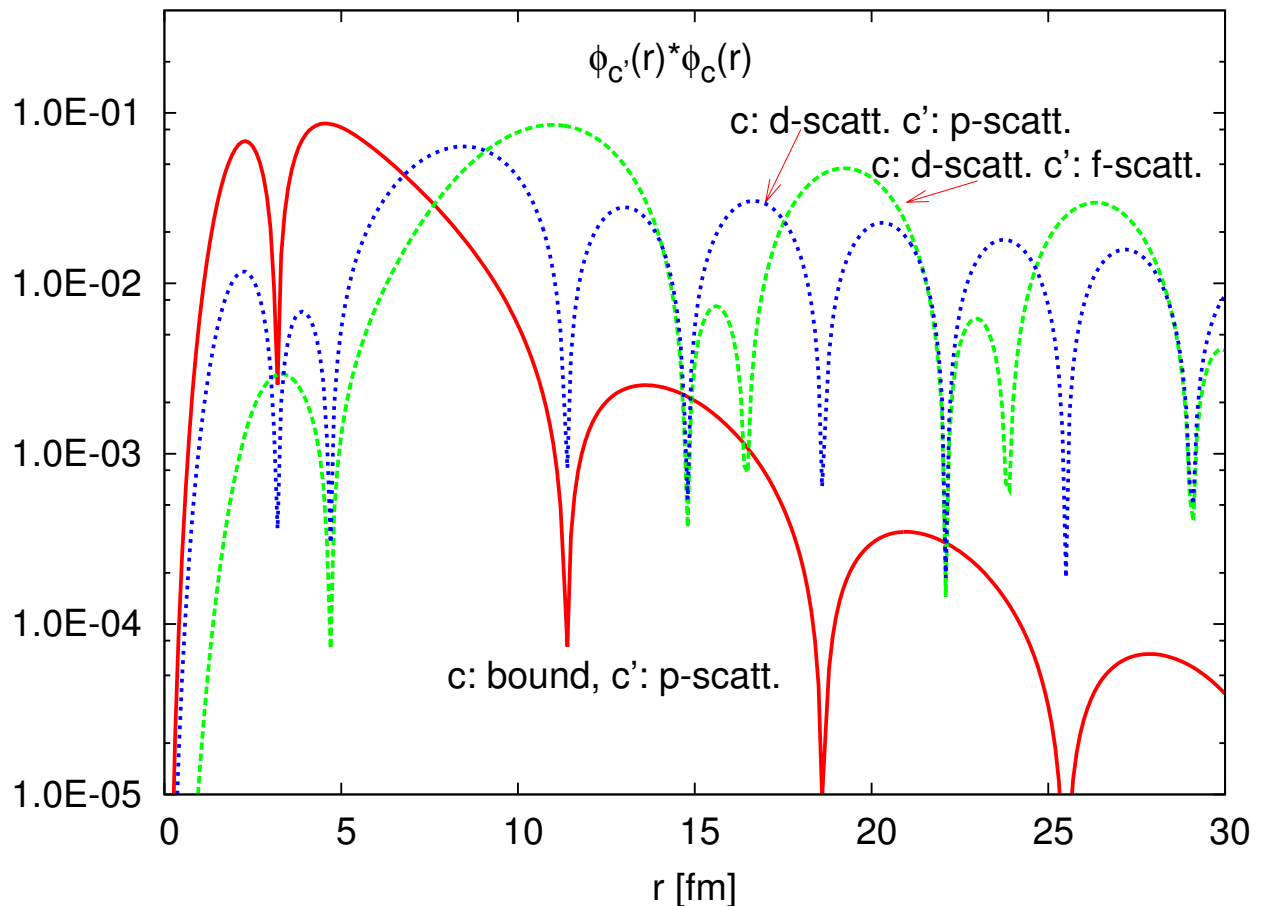
s , p , d -states have a node $r < 4 \text{ fm}$
 but not for f -state(No bond states)

product of $\hat{\phi}$
(bound) $\times$ (scatt.) state wf.



this product is related to
break up reaction

prod. of continuum $\hat{\phi}$'s



break up is induced at small r

but at large r

cont.-cont. coupling is important

and is called “post acceleration”

V_{13} and V_{23} :

OP pot. are used

$V_{(p-^{12}\text{C})}$ Watson et al.,

$V_{(^{22}\text{Mg}-^{12}\text{C})}$ Beunerd et al.,

↑ remains ambiguity

if we require $\hat{\phi}$ feels nucleus

$$r_{max} \geq \frac{m_{\text{Mg}} + m_p}{m_p} R_{max} !$$

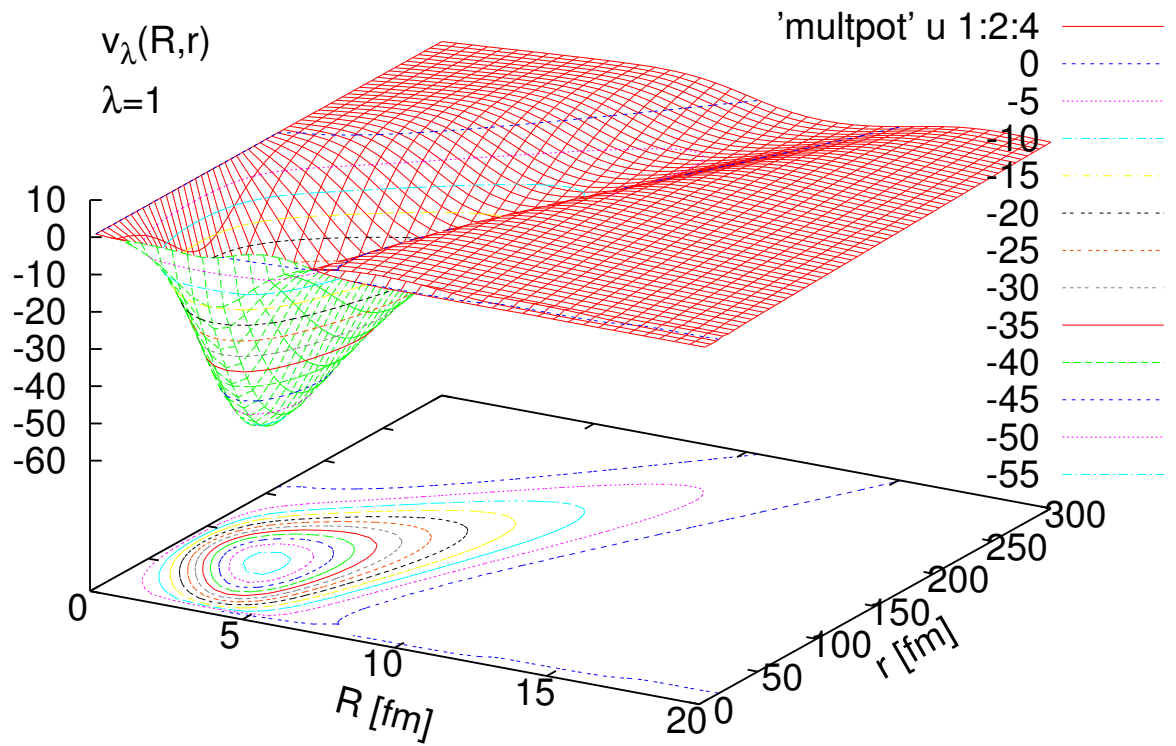
No excitation of ^{12}C nor ^{22}Mg

No coalescence

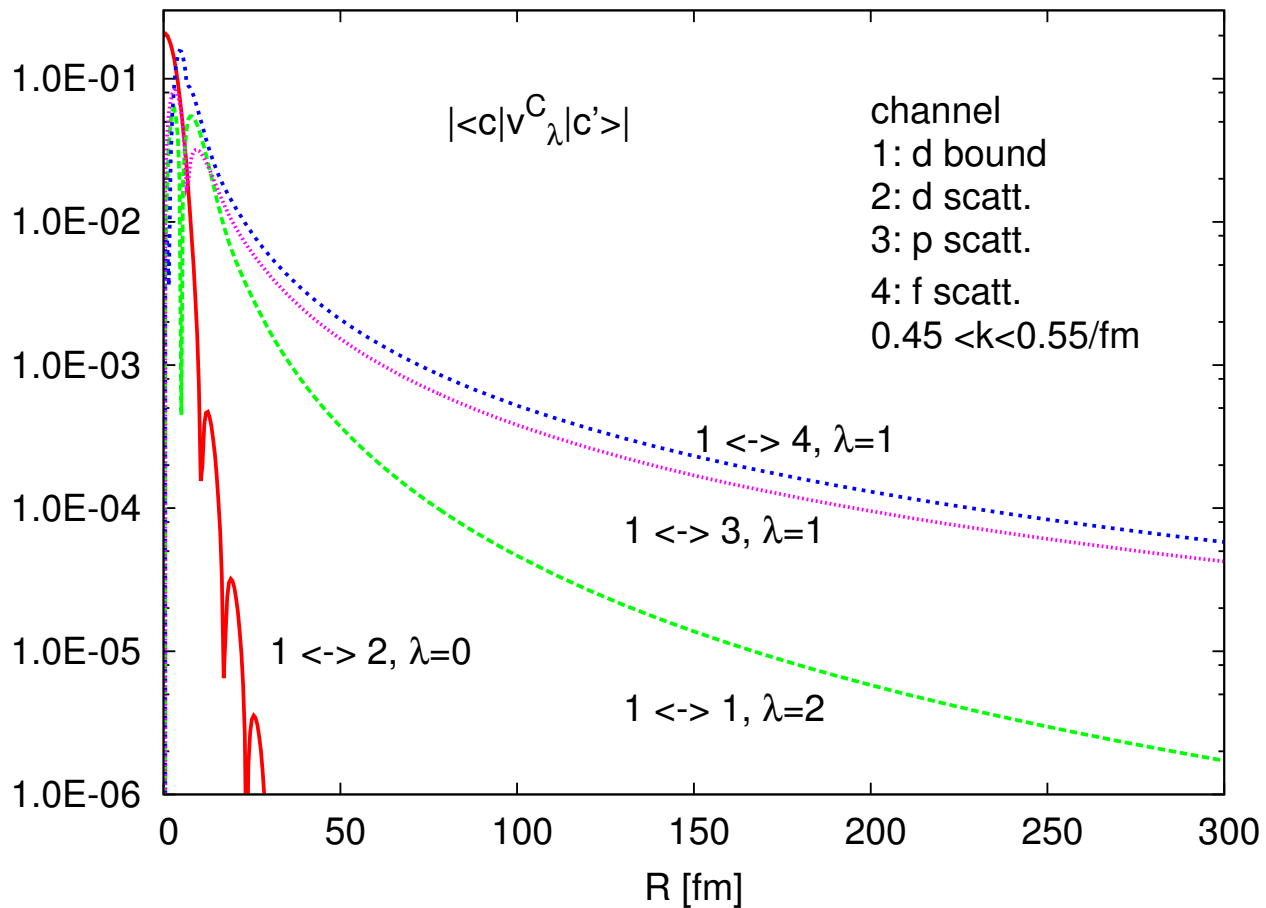
No Coul. BU included

in the final analysis !

dipole part of potential



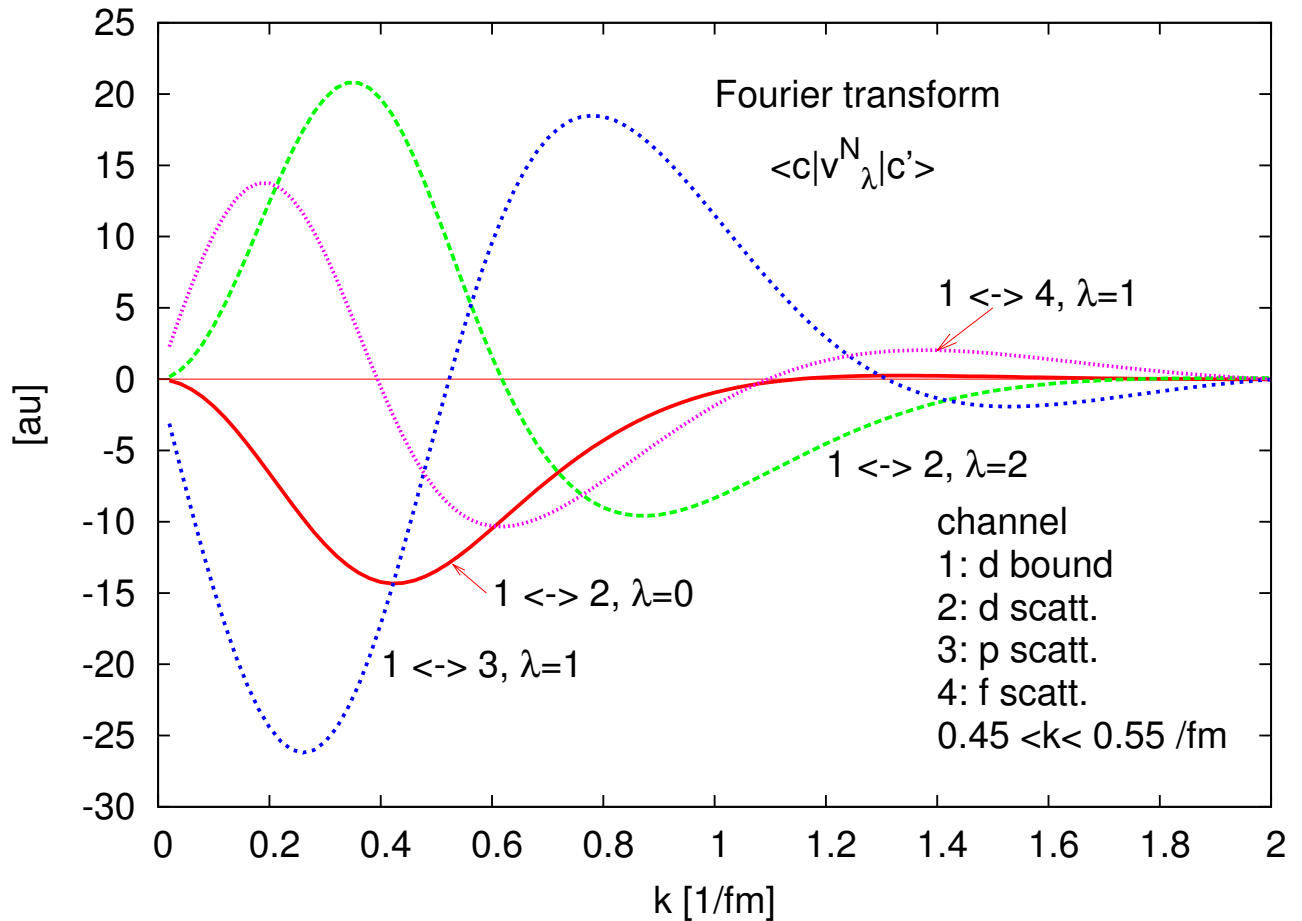
$$\langle \hat{\phi}_c(r) | v_\lambda^C(r, R) | \hat{\phi}_{c'}(r) \rangle_r$$



off diag. monopole elem. dumps
 very rapidly,
 due to orthogonality

$\lambda(> 0)$ elements dump as
 $R^{-(\lambda+1)}$ for large r

Fourier exp. of $\langle c | v_{\lambda}^N(r, R) | c' \rangle_r$

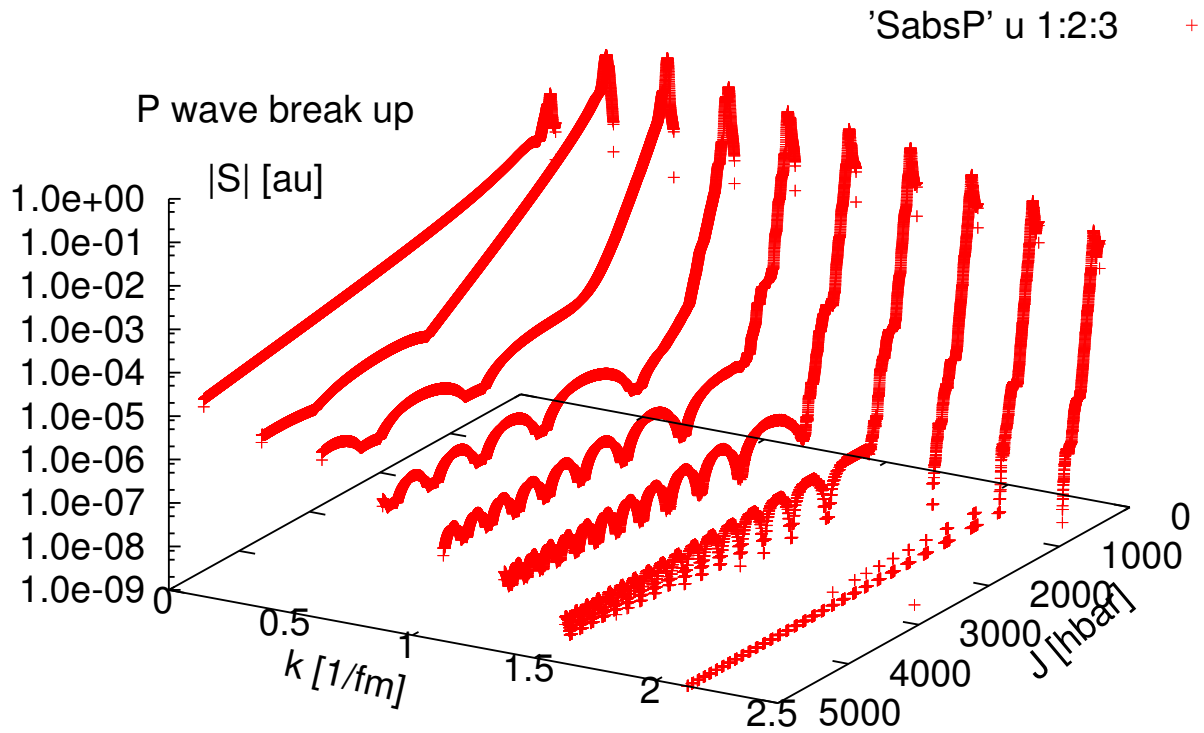


$\lambda = 0, 1$ and 2 component for

$c \leftrightarrow c'$ transition

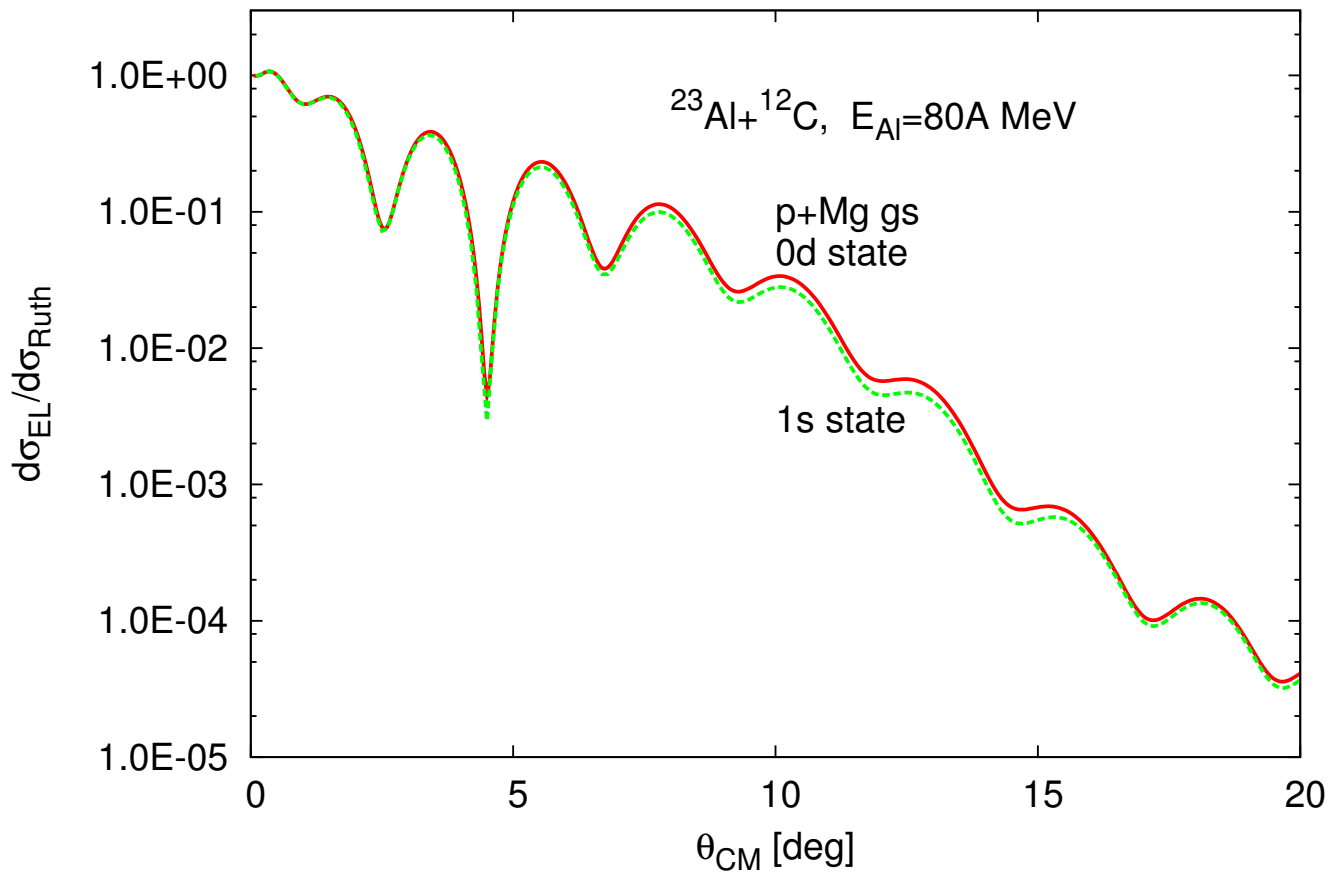
- * CDCC eq. solved numerically
by using 8 point Stömer's method
- * safeguard
integrate NOT from the origin
- * to get indep. sol. vectors
occasional ortho-normalization
- * S-mat.: cond. num. estimated
- * Coul. wf. Virmigham approach.
use of continued fraction
- * 3- 6-j: use of 3 term recur. relation
purge factorial evaluation

J and k dep. of *p*-wave absorption



just qualitative !

elastic scatt. cross sec.



$l=0$ to 4 states of 1-2 system

$$0 < k < 1.5 \text{ fm}^{-1}$$

No Coul. break up

$$\text{gs. of } ^{23}\text{Al} \begin{cases} \pi 0d \text{ state} \\ \pi 1s \text{ state} \end{cases}$$

No exp. data

triple diff. cross sec.

No Coul. BU included yet

s-, *d*-state for $p\text{-}^{22}\text{Mg}$ bound state

NOT symmetric about $k = 0$

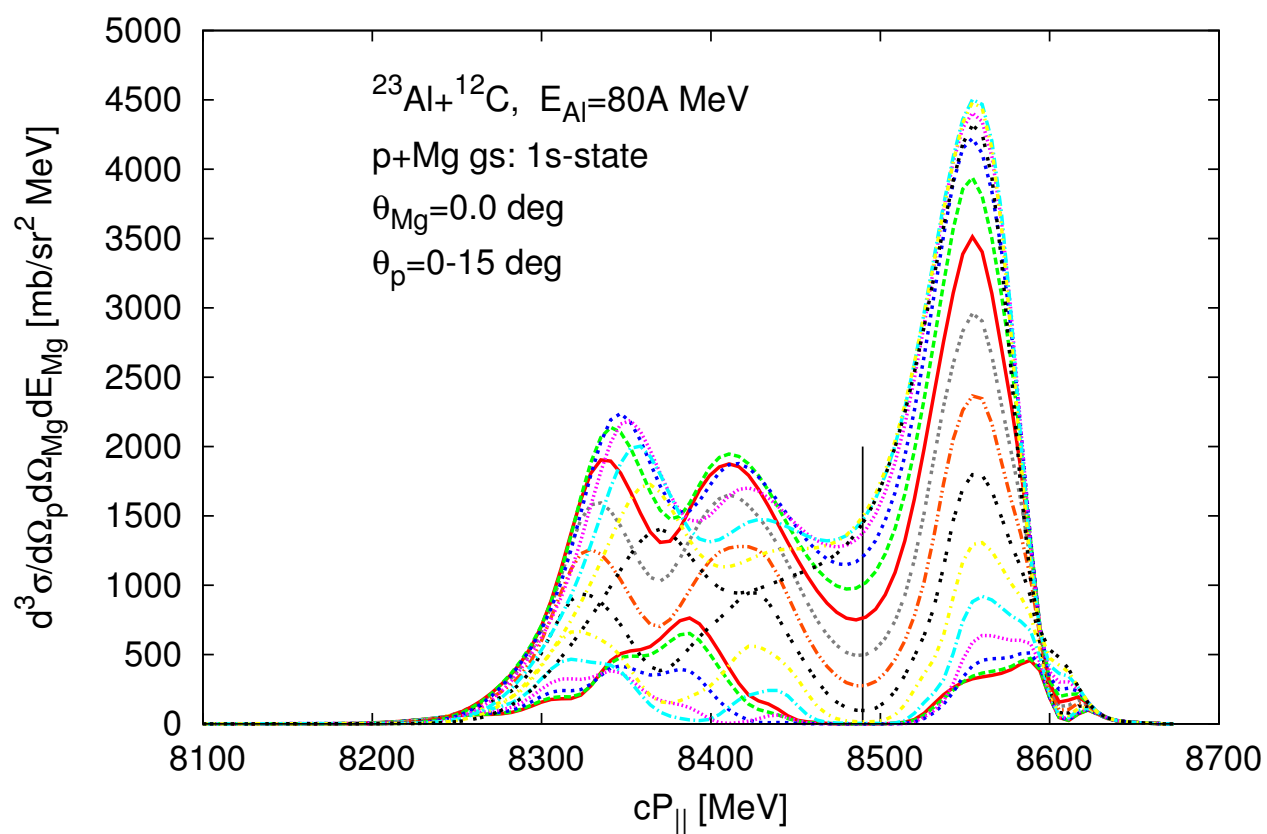
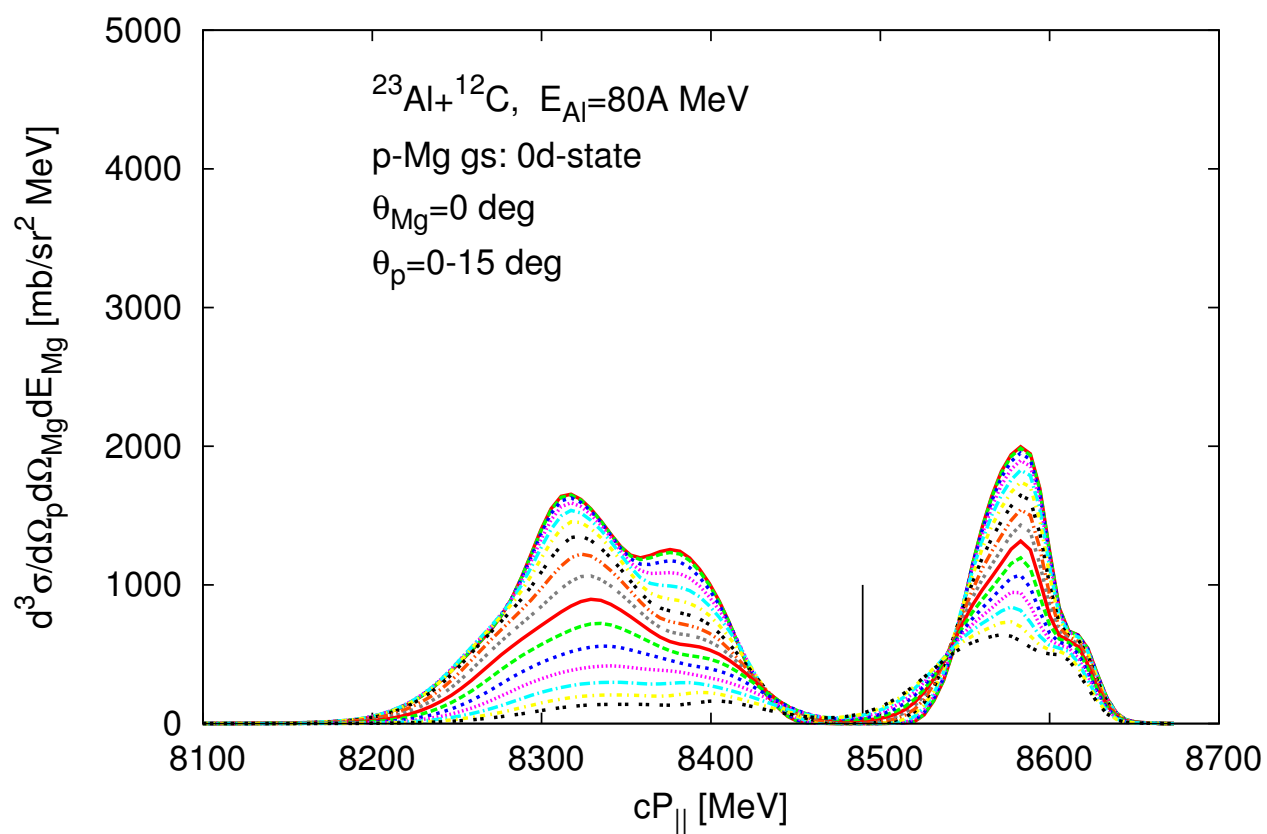
central suppression

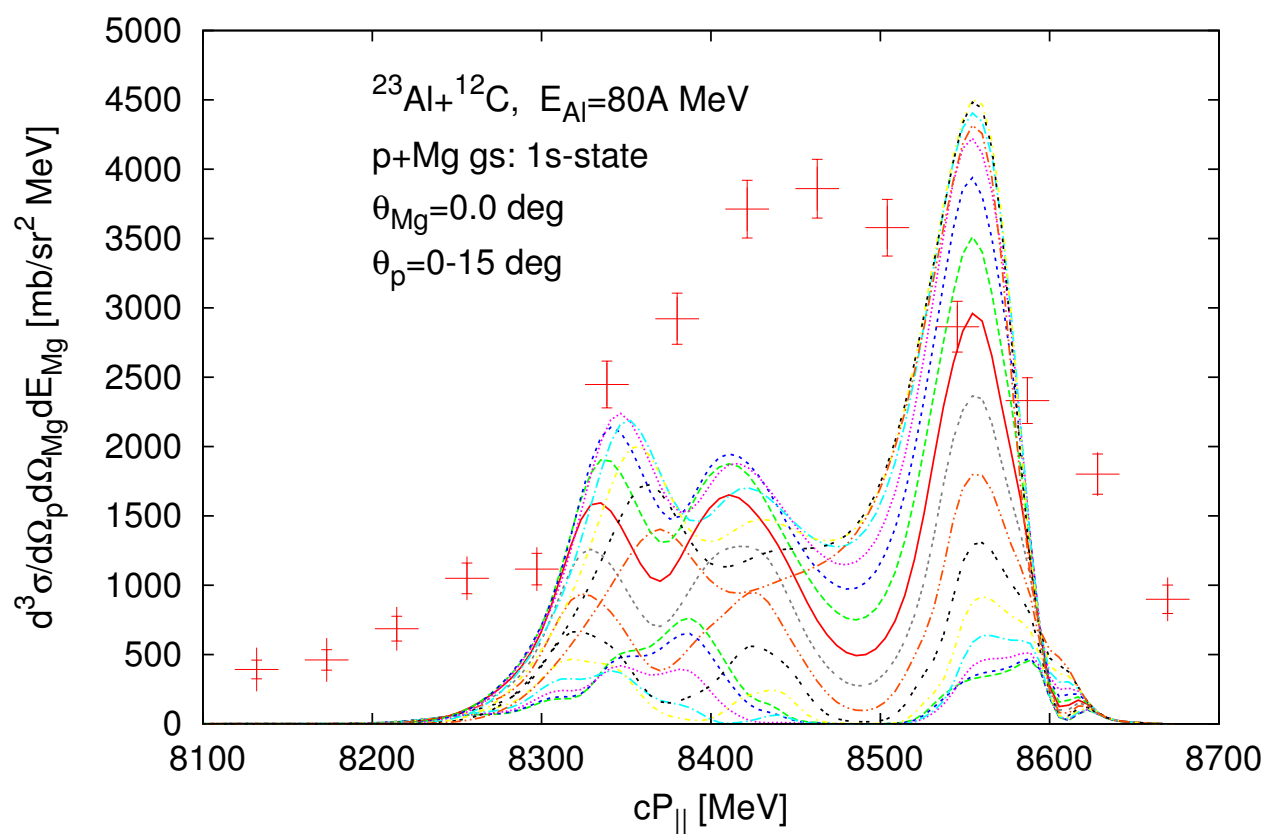
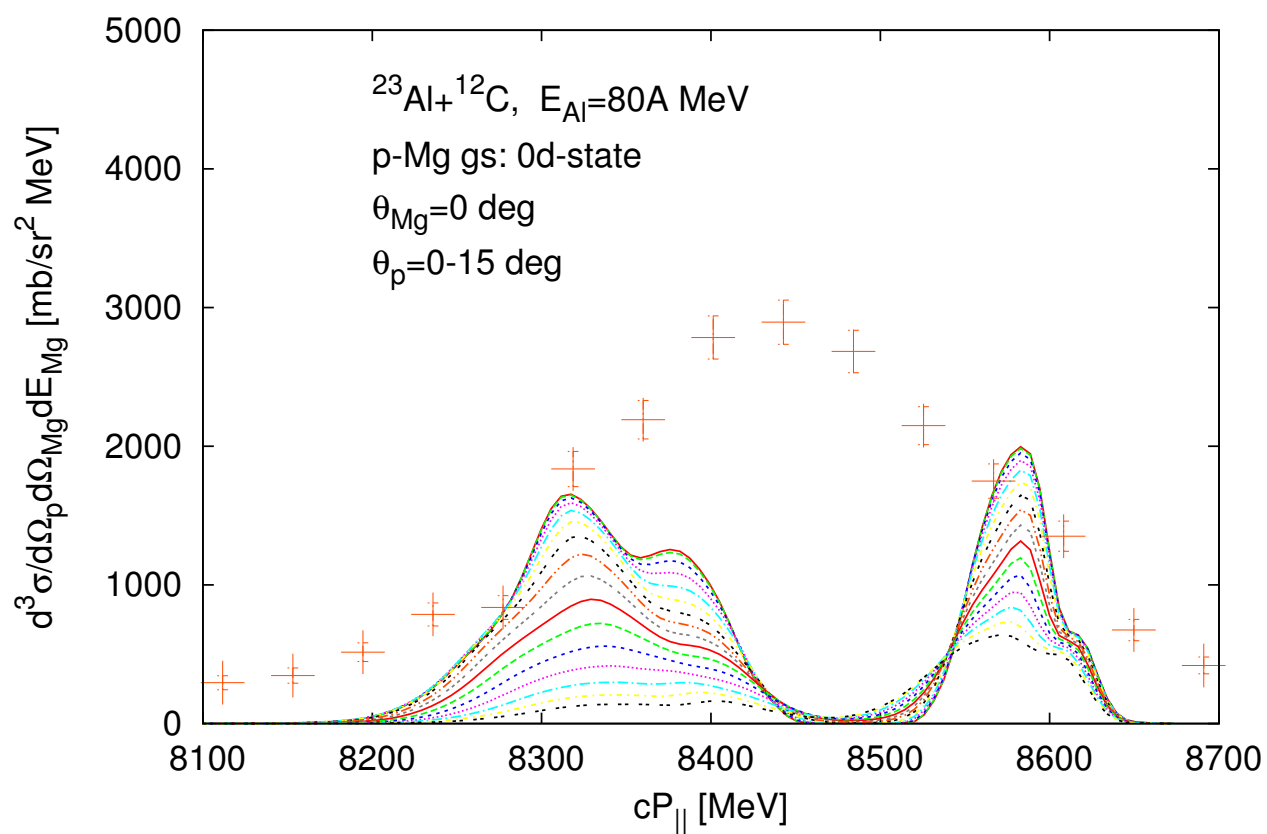
dipole break up dominates

Coulomb suppression of $\hat{\phi}$

Coul. BU may fill the dip

d-state be preferred for $p\text{+}^{22}\text{Mg}$ gs





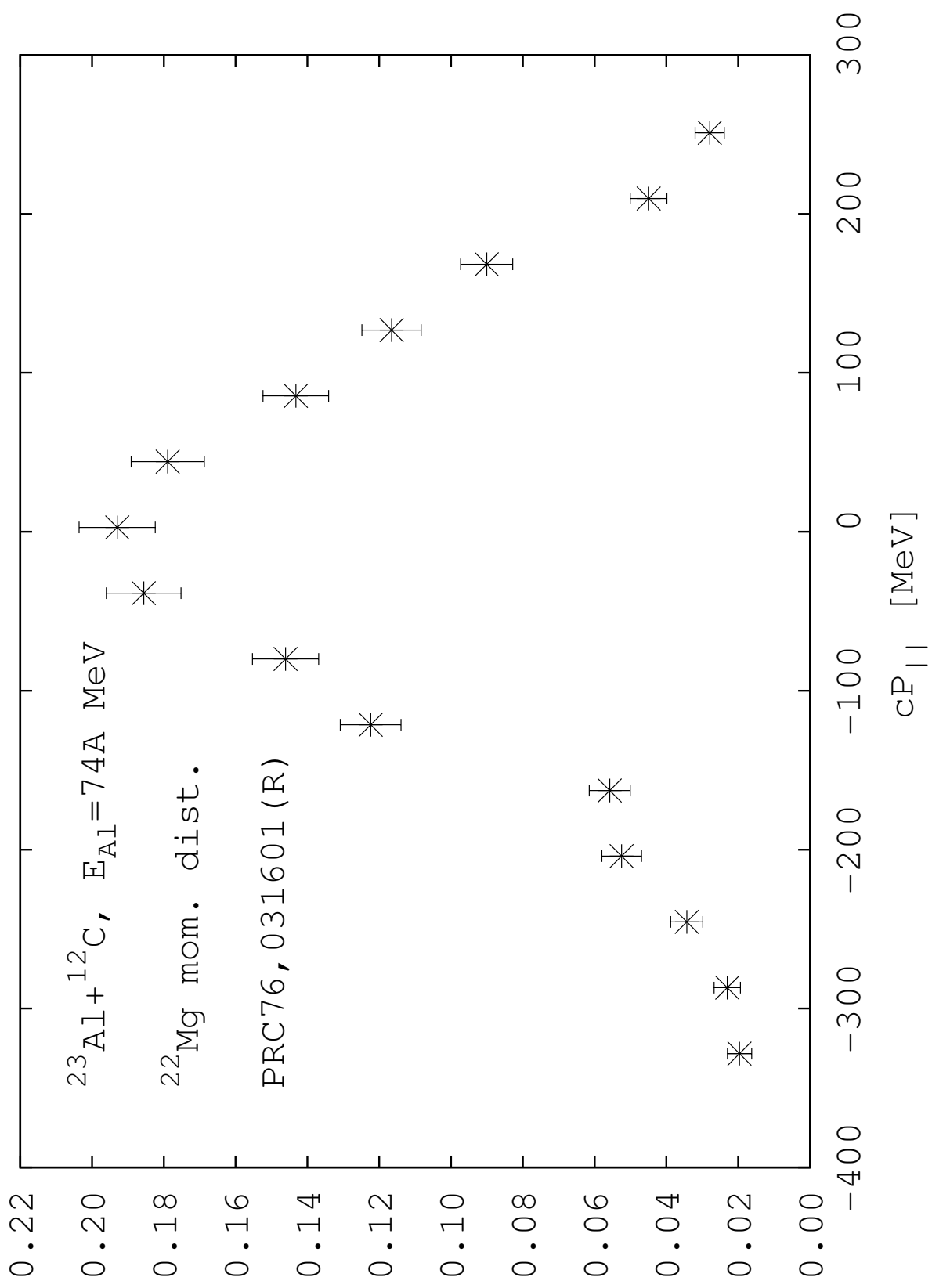
$$\begin{aligned}
& \text{expanding Coul. pot. } (R > r) \\
V_{13}^C(r_{13}) + V_{23}^C(r_{23}) & \propto \frac{z_1}{r_{13}} + \frac{z_2}{r_{23}} \\
& \sim \frac{z_1 + z_2}{R} - (\alpha z_1 + \beta z_2) \frac{r \cdot R}{R^3} \\
& \quad - \frac{\alpha^2 z_1 + \beta^2 z_2}{2R^3} \left\{ r^2 - 3 \left(\frac{r \cdot R}{R^2} \right)^2 \right\}
\end{aligned}$$

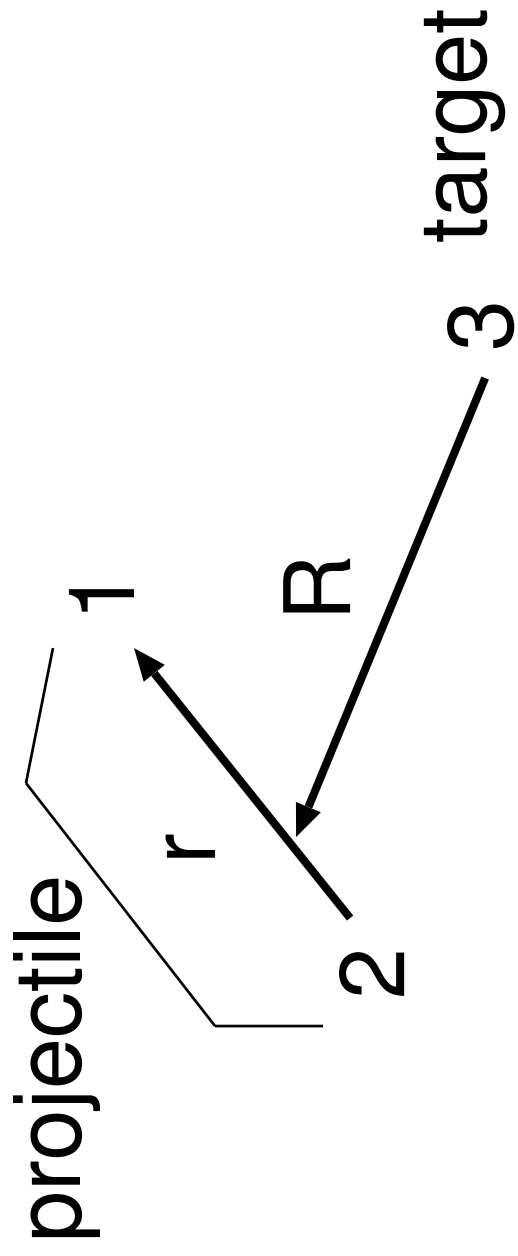
effective charge z_{eff} for CBU

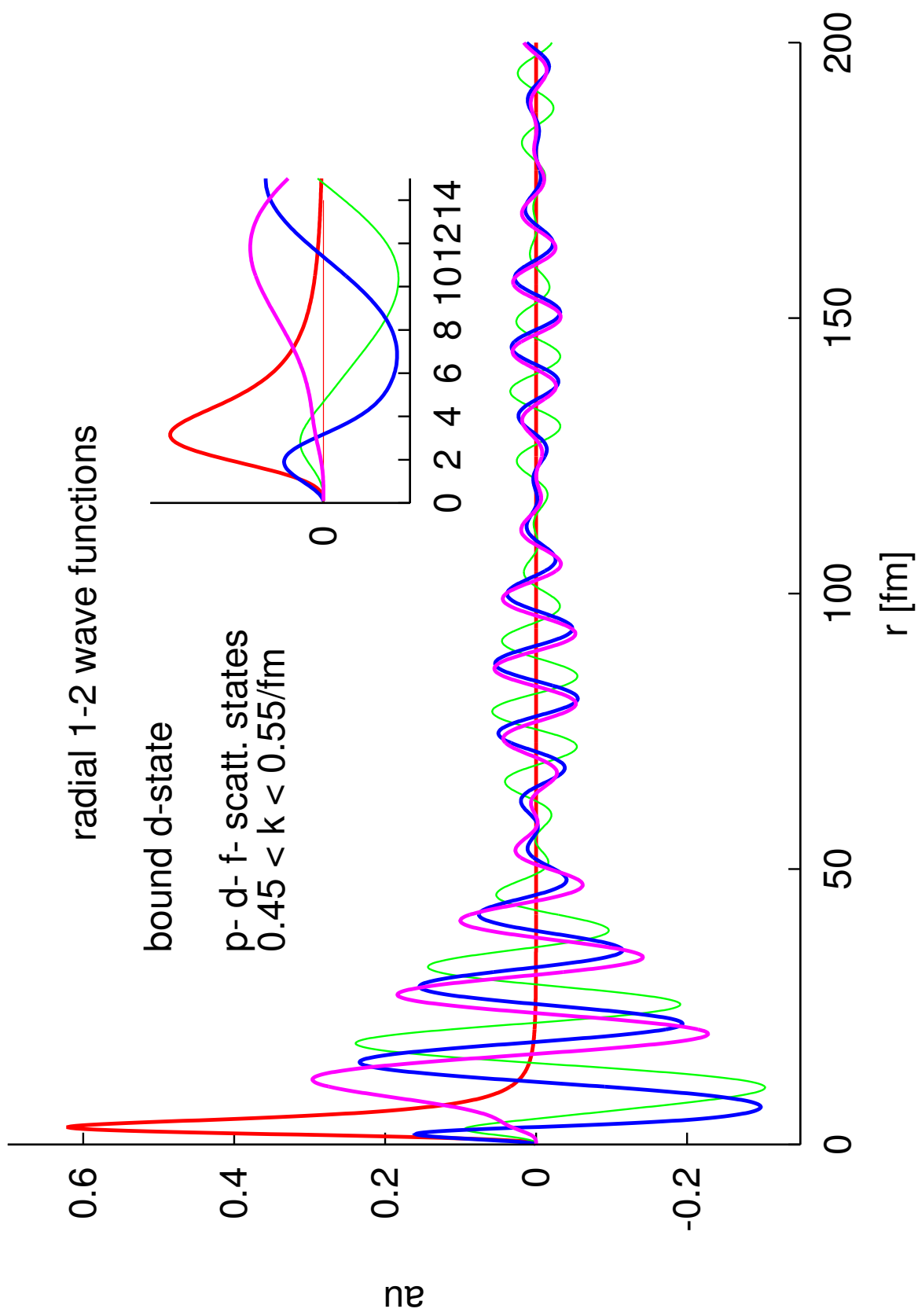
multipol.	z_{eff}	for $p+^{22}\text{Mg}$
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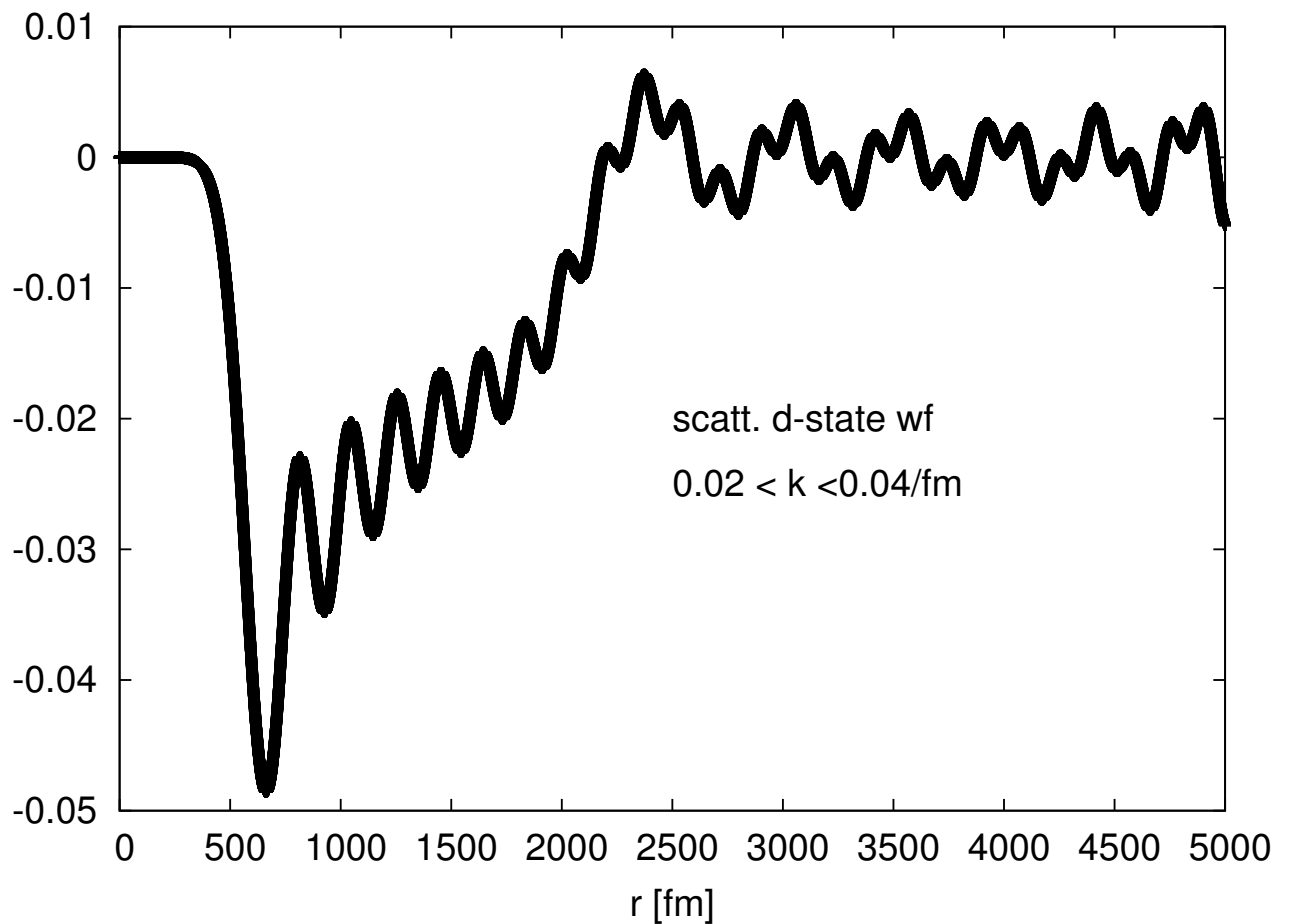
E1	$\frac{m_2 z_1 - m_1 z_2}{m_1 + m_2}$	0.438
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E2	$\frac{m_2^2 z_1 + m_1^2 z_2}{(m_1 + m_2)^2}$	0.917
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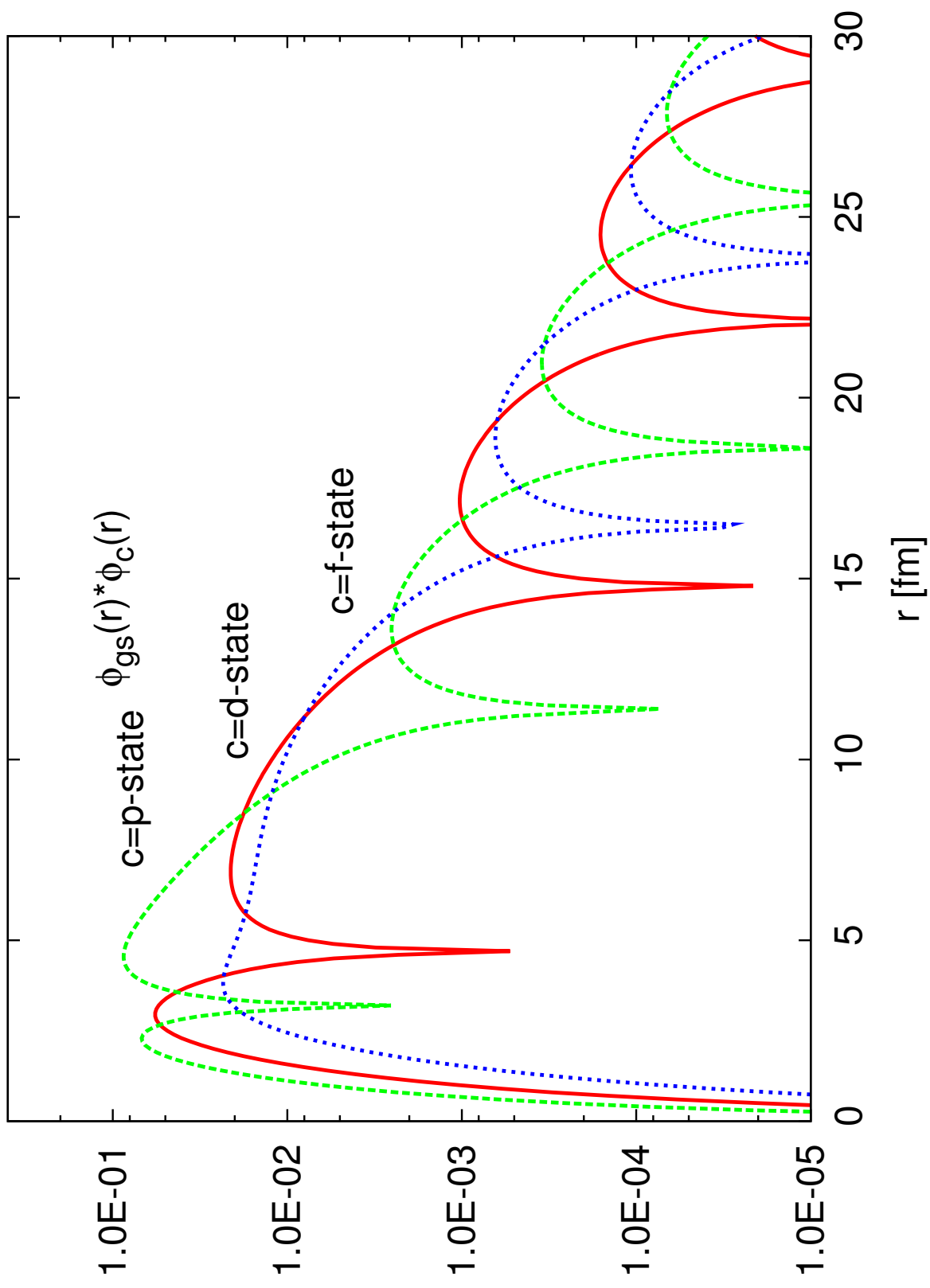


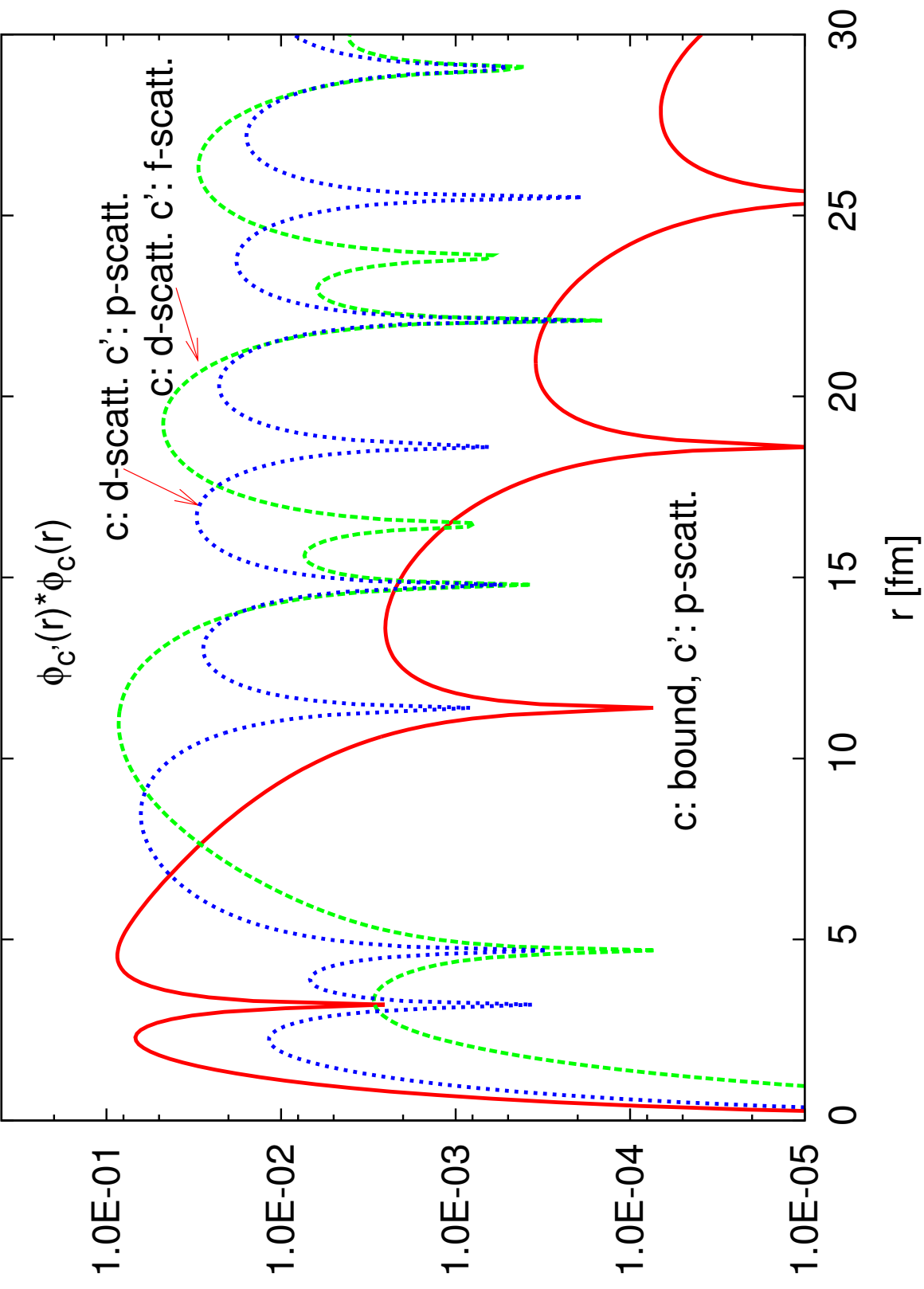


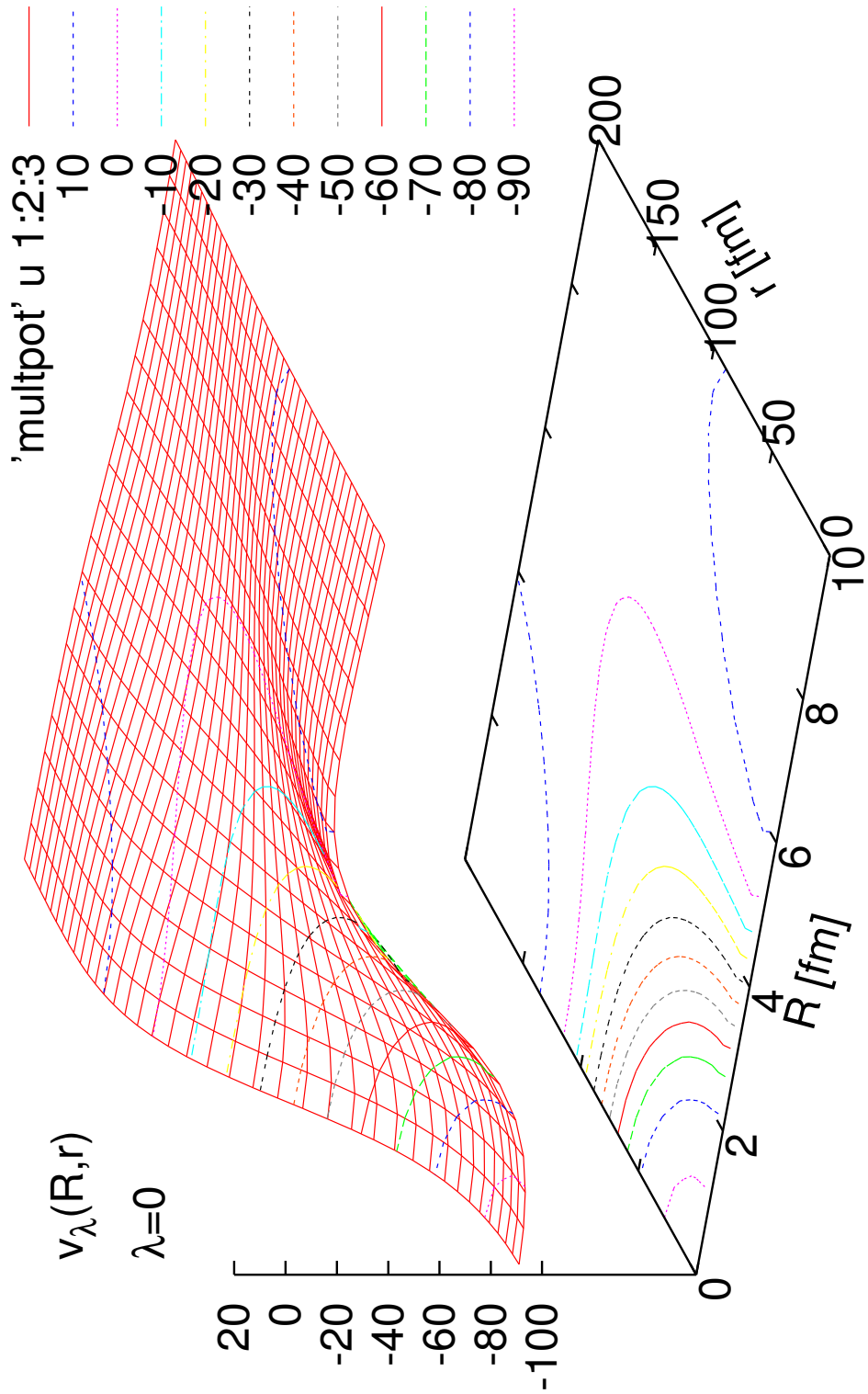
scatt. d-state wave function

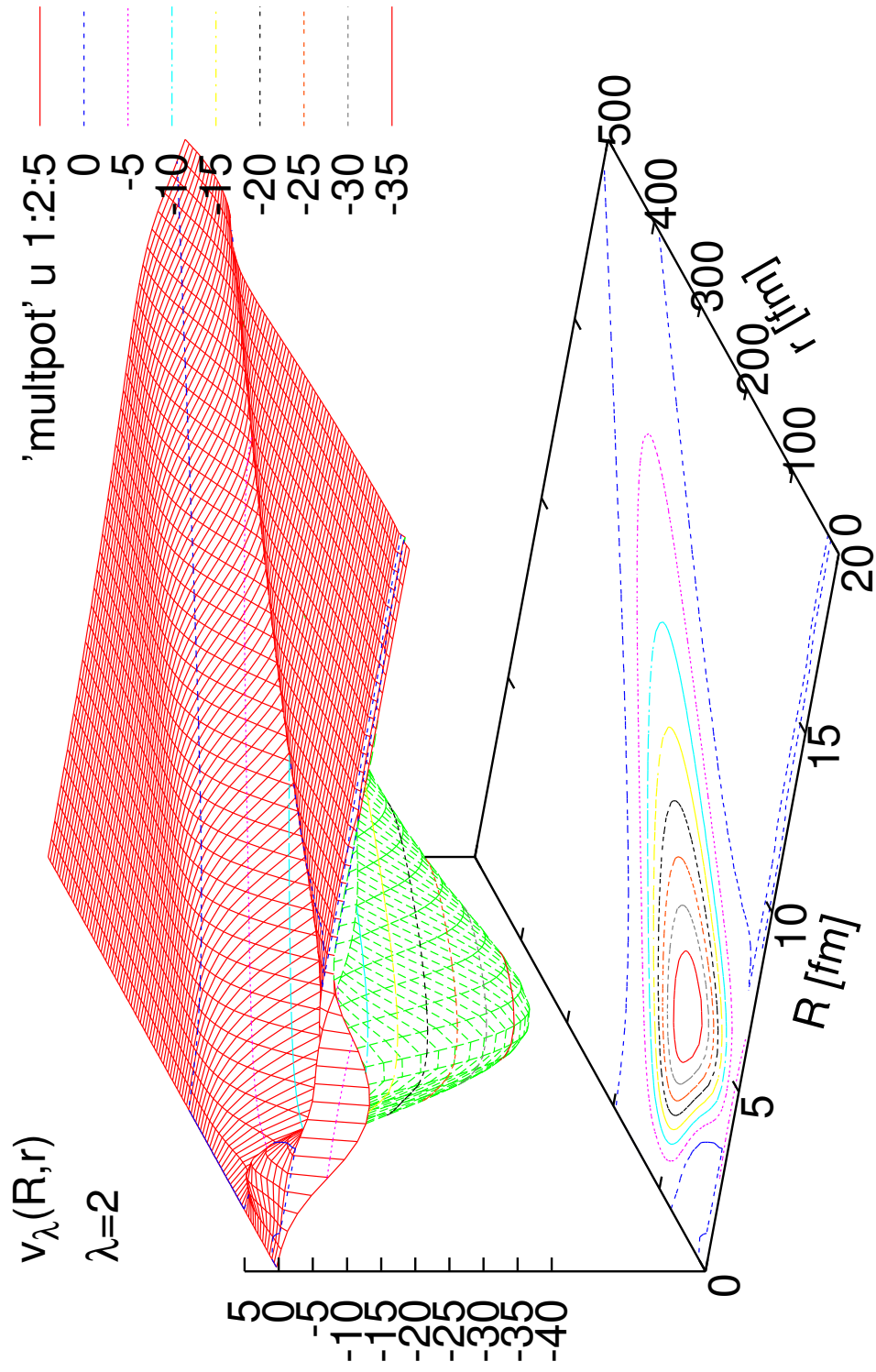
$$0.02 < k < 0.04/\text{fm}$$

vanishing amplitude near $r=0$, due to
Coulomb and centrifugal repulsion









$$\langle \hat{\phi}_c(r) | v_{\lambda=1}^C(r, R) | \hat{\phi}_{c'}(r) \rangle_r \propto R^{-2}$$

