TWOSTP

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TWOFNR

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0 Gross structure of the program TWOFNR

The program **TWOFNR** is the extended version of the program **TWOSTP** to calculate the finite-range form factor. This version includes all the functions of the program **TWOSTP**.

```
                              MAIN ( SKIP, CLEB, RAC, U9)
MIX      BASIC    FFGENZ   FFTAPE   FFGENF   ELAST    AMPCLA   AMPCLB   XSECTN
XSECTN   INPUTA   INPUTB            INPUTC   ELXSEC   OVLAPA   OVLAPA   OUTPUT
PLOT     SIGMA    FFSUB1            FFACT             BLOCK    BLOCK    PLOT
         COULFN   FFSUB4            LEGEND            OVLAPB   OVLAPB
         COUNEG   FFSUB6            GAUSS    LEGD     LEGD     LEGD
         PGEN2    BESSEL            BOUND    INTE2    INTE2    INTE2
         WRITEA   FFSUB7            FXGEN    FNSG2    FNSG2    FNSG2
                  FNLOC             CLEBZ
                  WRITEB            GK2GEN
                                    GBESSE
                                    GKGEN
                                    INTPOL
                                    WRITEC
```

Segment BASIC
reads the kinematical data, optical potential parameters and scattering angles and prepares Coulomb phase shift, Coulomb wave function and optical potential forms.
Subroutine MIX
reads the data of mixing factors of the scattering amplitudes, reads the sacattering amplitudes from the file, sums up coherently and calculates the cross section and polarization observables.
Segment FFGENZ
reads the data of the zero-range form factor and prepares the zero-range form factor, the correction factors for the nonlocality of the optical potentials and the finite-range correction factor for the zero-range form factor.
Subroutine FFTAPE
reads the finite-range form factor from the permanent file.
Segment FFGENF
reads the data of the finite-range form factor and prepares the finite-range form factor.
Segment ELAST
calculates the elastic scattering amplitudes, cross sections and polalizations.
Segment AMPCLA
calculates the DWBA overlap-integrals and the scattering amplitudes.
Subroutine AMPCLB
is the non spin-orbit version of the subroutine AMPCLA. When all of the optical potentials do not include the spin-orbit coupling, the flow goes through this routine.

Segment `XSECTN`

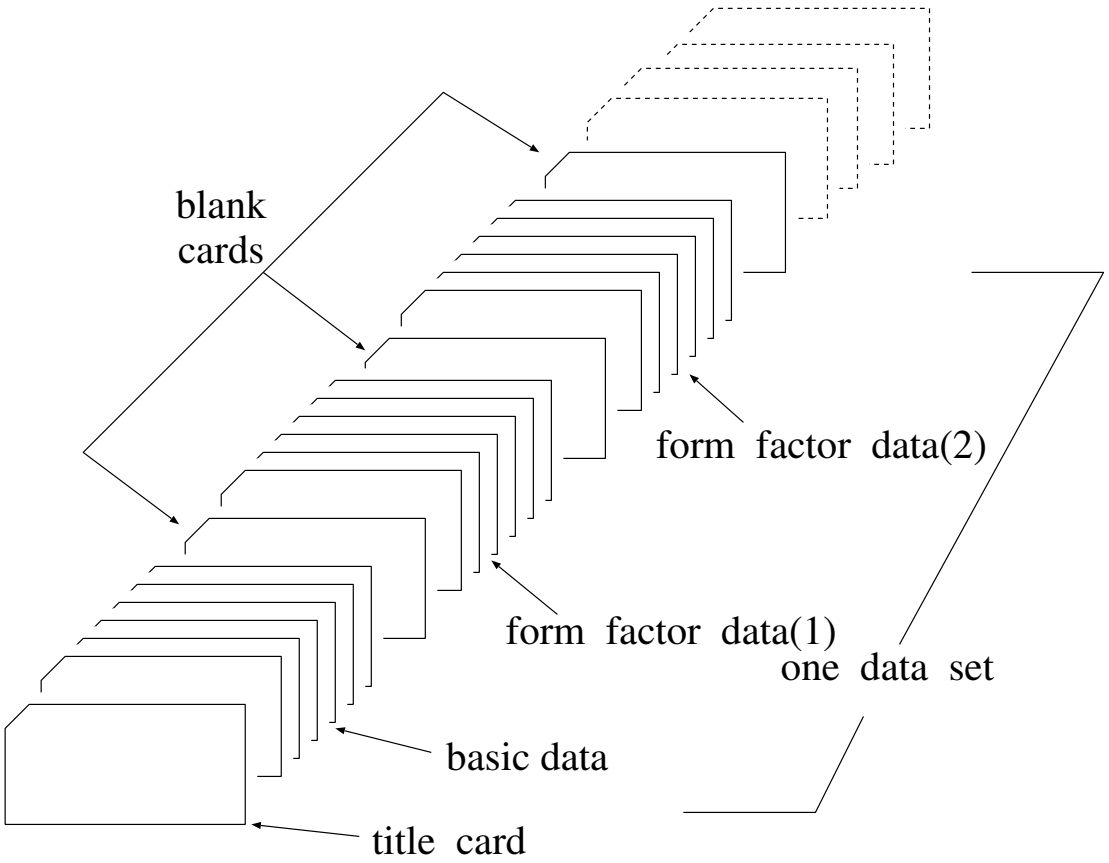
calculates the cross section and polarization observables, prints those and plots the cross section.

The files used in the program

I/O number	type of file	use
5		standard input unit
6		standard output unit
7		card punch
8	sequential permanent file	storage of scattering amplitudes
	Following files are used in the case of finite-range form factor	
1	sequential scratch file	storage of $g_k(r_a, r_b)$
2	direct access scratch file	storage of first-step form factor
3	direct access scratch file	storage of second or one-step form factor
12	sequential permanent file	storage of first-step form factor
13	sequential permanent file	storage of second or one-step form factor

1 Instructions for the Input Data

This program has two subroutines, which read input data.
INPUTA ; for Basic Data,
INPUTB ; for Form Factor Data,



One data set for one run consists of a set of "basic data" and some sets of "form factor data". Each set of basic data and form factor data is terminated by a blank card.

1.1 Basic Data

1.1.1 Title card

`FORMAT(10I1,3I2,I4,15A4)`

input variable	meaning
`KTOUT(1)`	Output of $\beta_{lsj}^{mm_b m_a}(\theta)$ =0; none =1; prints, =2; punches, =3; prints and punches, =4; prints and writes on file, =5; writes on file =6; writes $\beta_{el}^{m_a' m_a}(\theta)$ on file, =9; goes to the subroutine MIX.
`KTOUT(2)`	prints overlap integral =0; no, =1; yes.
`KTOUT(3)`	prints distorted wave of incident channel =0; no, =1; yes.
`KTOUT(4)`	prints distorted wave of exit channel =0; no, =1; yes.
`KTOUT(5)`	prints distorted wave of intermediate channel (1) =0; no, =1; yes.
`KTOUT(6)`	prints distorted wave of intermediate channel (2) =0; no, =1; yes.
`KTOUT(7)`	plots cross section =0; no, =1; yes.
`KTOUT(8)`	prints elastic cross section =0; no, =1; yes.
`KTOUT(9)`	is not used.
`KTOUT(10)`	is not used.
`INS`	ID number to identify $\beta(\theta)$, which is written on file.
`NUMRUN(1)`	Month.
`NUMRUN(2)`	Day.
`NUMRUN(3)`	Year.
`ITITOL(I)` `I=1,15`	Title to identify the run.

1.1.2 Numerical Data

Most of the numerical data are read in with `FORMAT(8F10.5)`. Integer numbers are also read in with the same FORMAT and converted to integer numbers in the program. Except for special cases, the first number of each input card identifies the group of data. The first decimal space of this number, which is indicated by "□", identifies the channel,
1 ; incident channel,
2 ; exit channel,
3 ; intermediate channel (1),
4 ; intermediate channel (2).

Hereafter one frame, which includes seven data at most corresponds to one input card (one data group). Only the "numerical data" cards relevant to the run should be read in. When several runs are sequentially continued, the "numerical data" card, which is exactly the same as that of the run just before, needs not be read in.

FORMAT(8F10.5)

1.0	
NUBCHN	Number of intermediate channels. (NUBCHN $\leq$ 2)
RMAX	Upper cut-off radius of the radial integrals. (fm)
NRMIN	Mesh point number of lower cut-off. $R_{cl} = \triangle r \times$NRMIN
NRMAX	Number of mesh points, $\triangle r$ =RMAX/NRMAX. (NRMAX$\leq$ 200)
ELABI	Laboratory energy of incident projectile. (MeV)
2.□	
TRS(1,□)	s_1 ; Spin transfer of the first step.
LTR(1,□)	l_1 ; Orbital angular momentum transfer of the first step.
TRJ(1,□)	j_1 ; Total spin transfer of the first step.
TRS(2,□)	s_2 ; Spin transfer of the second step.
LTR(2,□)	l_2 ; Orbital angular momentum transfer of the second step.
TRJ(2,□)	j_2 ; Total spin transfer in the second step.
TTRJ	j ; Total transferred angular momentum of the reaction.

For one-step process □ = 2. For the two-step process going through the intermediate channel (1), □ = 3. For the two-step process going through the intermediate channel (2), □ = 4. (The same for BETA(□) on the next card.) For one-step process the default for TTRJ is TRJ(1,□).

3.□	
LMIND(□)	Lower cut-off of partial wave.
LMAXD(□)	Maximum partial wave. (LMAXD(□) $\leq$ 80)
BETA(□)	Mixing factor of the process. If zero this process is not calculated.
PNLOC(□)	β ; Nonlocality parameter of the distorted wave.
4.□	
PMAS(□)	m_p ; Projectile mass.
TMAS(□)	M_T ; Target mass.
PZ(□)	z_p ; Projectile charge number.
TZ(□)	Z_T ; Target charge number.
PSPN(□)	s_p ; Projectile spin.
TSPN(□)	I_T ; Target spin.
QVLUE(□)	Q ; Q-value relative to the incident channel.

If any radius parameter r_x is positive, the radius is given by $R_x = r_x \times M_T^{1/3}$, and if it is negative, the radius is given by $R_x = |r_x| \times (M_T^{1/3} + m_p^{1/3})$

5.□	
VD(□)	V ; Real well depth of the optical potential.
WD(□)	W ; Imaginary well depth.
VSOD(□)	V_{so} ; Real well depth of spin-orbit term.
WSOD(□)	W_{so} ; Imaginary well depth of spin-orbit term.
RRD(□)	r_0 ; Real well radius parameter.
ARD(□)	a_r ; Real well diffuseness parameter.
RCD(□)	r_c ; Coulomb charge radius parameter.
6.□	
RSORD(□)	r_{sr} ; Real well radius parameter of spin-orbit term.
ASORD(□)	a_{sr} ; Real well diffuseness parameter of spin-orbit term.
RSOID(□)	r_{si} ; Imaginary well radius parameter of spin-orbit term.
ASOID(□)	a_{si} ; Imaginary well diffuseness parameter of spin-orbit term.

The form of the surface imaginary well is the first derivative of Woods-Saxon form, if $r_g = a_g = 0$ in the next card, otherwise it is Gaussian form.

7.□	
CSDGD(□)	C_{sd} ; Mixing factor of volume and surface imaginary well.
RID(□)	r_i ; Imaginary well radius parameter.
AID(□)	a_i ; Imaginary well diffuseness parameter.
RGD(□)	r_g ; Gaussian type imaginary well radius parameter.
AGD(□)	a_g ; Gaussian type imaginary well range parameter.
8.□	
KTISP(□)	Two figures integer for control of isospin term, tens (units) digit for real (imaginary) well, =0; none, =1; volume type, =2; surface type.
VISD(□)	V_{is} ; Real well depth of isospin term.
RISRD(□)	r_{isr} ; Real well radius parameter of isospin term.
AISRD(□)	a_{isr} ; Real well diffuseness parameter of isospin term.
WISD(□)	W_{is} ; Imaginary well depth of isospin term.
RISID(□)	r_{isi} ; Imaginary well radius parameter of isospin term.
AISID(□)	a_{isi} ; Imaginary well diffuseness parameter of isospin term.

Form of Optical Potential

$$
\begin{aligned}
U = \; & - \; \{V_c f_c^{(R)}(r) + iW_c f_c^{(I)}(r)\} \\
& + \; 2(\frac{\hbar}{m_\pi c})^2 \frac{1}{r}\frac{d}{dr}\{V_{so} f_{so}^{(R)}(r) + iW_{so} f_{so}^{(I)}(r)\}\ \mathbf{l}\cdot\mathbf{s} \\
& - \; \{V_{is} f_{is}^{(R)}(r) + iW_{is} f_{is}^{(I)}(r)\}\ \mathbf{T}\cdot\boldsymbol{\tau} \\
& + \; \begin{cases} \frac{1}{2R_c}\left(3 - \frac{r^2}{R_c^2}\right) zZe^2 & r \le R_c \\ \frac{1}{r} zZe^2 & r \ge R_c, \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
f_c^{(R)}(r) &= \frac{1}{1+\exp[(r-R_R)/a_r]} \\
f_c^{(I)}(r) &= (1-C_{sd})\ \frac{1}{1+\exp[(r-R_I)/a_i]} + 4C_{sd}\ \frac{\exp[(r-R_I)/a_i]}{\{1+\exp[(r-R_I)/a_i]\}^2} \\
\text{or} \quad &= (1-C_{sd})\ \frac{1}{1+\exp[(r-R_I)/a_i]} + C_{sd}\exp\{-[(r-R_G)/a_g]^2\} \\
f_{so}^{(R)}(r) &= \frac{1}{1+\exp[(r-R_{SR})/a_{sr}]} \\
f_{so}^{(I)}(r) &= \frac{1}{1+\exp[(r-R_{SI})/a_{si}]} \\
f_{is}^{(R)}(r) &= \frac{1}{1+\exp[(r-R_{ISR})/a_{isr}]} \\
\text{or} \quad &= \frac{R_{ISR}}{a_{isr}}\frac{\exp[(r-R_{ISR})/a_{isr}]}{\{1+\exp[(r-R_{ISR})/a_{isr}]\}^2} \\
f_{is}^{(I)}(r) &= \frac{1}{1+\exp[(r-R_{ISI})/a_{isi}]} \\
\text{or} \quad &= \frac{R_{ISI}}{a_{isi}}\frac{\exp[(r-R_{ISI})/a_{isi}]}{\{1+\exp[(r-R_{ISI})/a_{isi}]\}^2}
\end{aligned}
$$

where for any x

$$
R_X = \begin{cases} r_x \times M_T^{1/3} & \text{if } r_x > 0 \\ |r_x| \times (M_T^{1/3} + m_p^{1/3}) & \text{if } r_x < 0. \end{cases}
$$

`9.0`
`JMAX`
`DTHETA`
`THETAD(1)`
	Number of angles. (JMAX $\le$ 73)
	$\triangle\theta$;Angle increment. If $\triangle\theta$=0, next cards(angles) should follow.
	θ_1 ; First angle.

`THETAD(J)` `J=1,JMAX`	Angles. (Center of Mass System)

A blank card will terminate the basic data.

blank card

1.2 Form Factor Data

One must input necessary number of data sets for the form factors in definite order. (The form factor data for the step of BETA(□)=0.0 must not be read in.)
Order;

1	One-step process.
2	First-step of the two-step process going through the intermediate channel(1).
3	Second-step of the two-step process going through the intermediate channel(1).
4	First-step of the two-step process going through the intermediate channel(2).
5	Second-step of the two-step process going through the intermediate channel(2).

In some cases, a coherent sum of form factors must be used, for example in two-nucleon transfer or microscopic inelastic excitation. If KTFF(4) in the next card is not zero, the program will attempt to read in following form factor data and add coherently to the previous ones, until a KTFF(4)=0 is detected which indicates the last set for this step. The data cards, which include only the same data as the data set just before, need not be read in.

`FORMAT(8F10.5)`

`10.00`	
`AMP`	a_{lsj} ; Spectroscopic amplitude.
`KTFF(4)`	Superposition control of form factor, =0; last case, =1; to be continued.
`D02`	D_0^2 ; Zero-range constant for transfer reaction.
`KTFF(5)`	Output control of form factor. =0; no, =1; prints =2; punches, =3; prints and punches.
`FTS`	τ ; Isospin transfer.
`TA`	T_A ; Target isospin of the prior channel.
`TB`	T_B ; Target isospin of the post channel.

Default for AMP=0.0 and D02=0.0 are AMP=1.0 and D02=1.018×10^4 respectively.

`10.01`	
`FNRNG`	R ; Finite-range parameter.

Collective form factor (FFSUB1)

10.11	
KTFF(2)	Control integer of real part. $0 \sim 11$.
FREAD(1)	V_0
FREAD(2)	r_0 $\quad R_x = r_x \times M_T^{1/3}$, if $r_x > 0$
FREAD(3)	a_0 $\quad R_x = \lvert r_x \rvert \times (M_T^{1/3} + m_p^{1/3})$, if $r_x < 0$
FREAD(4)	r_c
FREAD(5)	r_{ce}
10.12	
KTFF(3)	Control integer of imaginary part. $0 \sim 10$.
FREAD(6)	W_0
FREAD(7)	r_i $\quad R_x = r_x \times M_T^{1/3}$, if $r_x > 0$
FREAD(8)	a_i $\quad R_x = \lvert r_x \rvert \times (M_T^{1/3} + m_p^{1/3})$, if $r_x < 0$
FREAD(9)	r_g
FREAD(10)	b_g
FREAD(11)	C_{sd}

In the case $z_a \neq z_b$, for example (p,n) or (^{3}He,t) reaction process, the Lane potential is assumed, and the strength is multiplied by the factor $\sqrt{2T_A/2M_A}$, and further if the transferred $l = 0$, the strength is multiplied by a factor $\sqrt{4\pi}$, because $Y_0^0 = 1/\sqrt{4\pi}$.

Options available for collective excitation form factor

Real Part

`KTFF(2)`

$$
\begin{array}{lrcl}
0 & f^{(R)}(r) & = & 0 \\
1 & f^{(R)}(r) & = & \dfrac{R_0 V_0}{a} f_1^{(R)}(r) + C_l(r) \\
2 & f^{(R)}(r) & = & \dfrac{R_0 V_0}{a} \dfrac{r}{R_0} f_1^{(R)}(r) + C_l(r) \\
3 & f^{(R)}(r) & = & \dfrac{R_0 V_0}{a} f_1^{(R)}(r) \\
4 & f^{(R)}(r) & = & \dfrac{R_0 V_0}{a} \dfrac{r}{R_0} f_1^{(R)}(r) \\
5 & f^{(R)}(r) & = & -4 \dfrac{R_0 V_0}{a} f_2^{(R)}(r) \\
6 & f^{(R)}(r) & = & -4 \dfrac{R_0 V_0}{a} \dfrac{r}{R_0} f_2^{(R)}(r) \\
7 & f^{(R)}(r) & = & \dfrac{R_0^2 V_0}{a^2} f_2^{(R)}(r) \\
8 & f^{(R)}(r) & = & \dfrac{R_0^2 V_0}{a^2} \left\{ \dfrac{r}{R_0} \left[\dfrac{a}{R_0} f_1^{(R)}(r) + \dfrac{r}{R_0} f_2^{(R)}(r) \right] \right\} \\
9 & f^{(R)}(r) & = & V_0 f_0^{(R)}(r) \\
10 & f^{(R)}(r) & = & -4 V_0 f_1^{(R)}(r) \\
11 & f^{(R)}(r) & = & V_0 g_0^{(R)}(r)
\end{array}
$$

where

$$
\begin{array}{rcl}
C_l(r) & = & \dfrac{3 z_a Z_A e^2}{2l+1} \begin{cases} R_c^l / r^{l+1} & r \geq R_c \\ r^l / R_c^{l+1} & r \leq R_c \end{cases} \\
 & = & 0 \qquad\qquad r \leq R_{ce} \\
\texttt{BSUBL} & = & \dfrac{3 z_a Z_A e^2}{2l+1} = \dfrac{4.32\ z_a Z_A}{2l+1}
\end{array}
$$

$$
f_0^{(R)}(r) = [1 + \exp(X_R)]^{-1}, \qquad f_1^{(R)}(r) = \frac{d}{dX_R} f_0^{(R)}(r)
$$

$$
f_2^{(R)}(r) = \frac{d^2}{dX_R^2} f_0^{(R)}(r), \qquad g_0^{(R)}(r) = \exp[-X_R^2]
$$

$$
X_R = \frac{r - R_0}{a}, \qquad R_0 = \begin{cases} r_0 \times M_T^{1/3} & r_0 > 0 \\ |r_0| \times (M_T^{1/3} + m_p^{1/3}) & r_0 < 0 \end{cases}
$$

Imaginary Part

`KTFF(3)`

$$
\begin{aligned}
0 \quad & f^{(I)}(r) = 0 \\
1 \quad & f^{(I)}(r) = \frac{R_I W_0}{a_i}\left[(1-C_{sd}) f_1^{(I)}(r) - 4C_{sd} f_2^{(I)}(r)\right] \\
2 \quad & f^{(I)}(r) = \frac{R_I W_0}{a_i}(1-C_{sd}) f_1^{(I)}(r) + \frac{R_G W_0}{b_g} C_{sd} g_1^{(I)}(r) \\
3 \quad & f^{(I)}(r) = \frac{R_I W_0}{a_i}\left[(1-C_{sd})\frac{r}{R_I} f_1^{(I)}(r) - 4C_{sd}\frac{r}{R_I} f_2^{(I)}(r)\right] \\
4 \quad & f^{(I)}(r) = \frac{R_I W_0}{a_i}(1-C_{sd})\frac{r}{R_I} f_1^{(I)}(r) + \frac{R_G W_0}{b_g} C_{sd}\frac{r}{R_I} g_1^{(I)}(r) \\
5 \quad & f^{(I)}(r) = \frac{R_I^2 W_0}{a_i^2}\left[(1-C_{sd}) f_2^{(I)}(r) - 4C_{sd} f_3^{(I)}(r)\right] \\
6 \quad & f^{(I)}(r) = \frac{R_I^2 W_0}{a_i^2}(1-C_{sd}) f_2^{(I)}(r) + \frac{R_G^2 W_0}{b_g^2} C_{sd} g_2^{(I)}(r) \\
7 \quad & f^{(I)}(r) = \frac{R_I^2 W_0}{a_i^2}\Big\{(1-C_{sd})\frac{r}{R_I}\left[\frac{a_i}{R_I} f_1^{(I)}(r) + \frac{r}{R_I} f_2^{(I)}\right] \\
& \quad - 4C_{sd}\frac{r}{R_I}\left[\frac{a_i}{R_I} f_2^{(I)}(r) + \frac{r}{R_I} f_3^{(I)}\right]\Big\} \\
8 \quad & f^{(I)}(r) = \frac{R_I^2 W_0}{a_i^2}(1-C_{sd})\frac{r}{R_I}\left[\frac{a_i}{R_I} f_1^{(I)}(r) + \frac{r}{R_I} f_2^{(I)}\right] \\
& \quad + \frac{R_G^2 W_0}{b_g^2} C_{sd}\frac{r}{R_I}\left[\frac{b_g}{R_I} g_1^{(I)}(r) + \frac{r}{R_I} g_2^{(I)}(r)\right] \\
9 \quad & f^{(I)}(r) = W_0\left[(1-C_{sd}) f_0^{(I)}(r) - 4C_{sd} f_1^{(I)}(r)\right] \\
10 \quad & f^{(I)}(r) = W_0\left[(1-C_{sd}) f_0^{(I)}(r) + C_{sd} g_0^{(I)}(r)\right]
\end{aligned}
$$

where

$$
\begin{aligned}
f_0^{(I)}(r) &= [1+\exp(X_I)]^{-1}, & g_0^{(I)}(r) &= \exp[-X_G^2] \\
f_1^{(I)}(r) &= \frac{d}{dX_I} f_0^{(I)}(r), & g_1^{(I)}(r) &= \frac{d}{dX_I} g_0^{(I)}(r) \\
f_2^{(I)}(r) &= \frac{d^2}{dX_I^2} f_0^{(I)}(r), & g_2^{(I)}(r) &= \frac{d^2}{dX_I^2} g_0^{(I)}(r) \\
f_3^{(I)}(r) &= \frac{d^3}{dX_I^3} f_0^{(I)}(r) \\
X_I &= \frac{r-R_I}{a_i}, & X_G &= \frac{r-R_G}{b} \\
R_X &= \begin{cases} r_x \times M_T^{1/3} & r_x > 0 \\ |r_x| \times (M_T^{1/3} + m_p^{1/3}) & r_x < 0 \end{cases}
\end{aligned}
$$

Single-particle transfer form factor(FFSUB4)

10.41	
IREAD(1)	N ; Number of nodes excluding the origin and infinity.
IREAD(2)	zZ ; Charge product of bound particle and core.
FREAD(1)	E ; Binding energy (MeV; must be positive).
FREAD(2)	m ; Mass of bound particle.
FREAD(3)	M ; Mass of core.
FREAD(4)	$\triangle$M (= 0, in usual case.)
FREAD(5)	$\triangle$Z (= 0, in usual case.)
10.42	
FREAD(6)	r_0 ; Well radius parameter. $R_0 = r_0 \times M^{1/3}$
FREAD(7)	r_{cn} ; Coulomb charge radius parameter. $R_c = r_{cn} \times M^{1/3}$
FREAD(8)	a_n ; Well diffuseness.
FREAD(9)	V_{son} ; Depth of the spin-orbit well.
FREAD(23)	β ; Nonlocality range parameter.

External form factor Reads in the real part of form factor with FORMAT(8F10.5)

10.51	
FF(I) I=1,NRMAX	Real part of form factor.

Reads in the imaginary part of form factor with FORMAT(8F10.5)

10.52	
FFI(I) I=1,NRMAX	Imaginary part of form factor.

Reads in the real part of form factor with FORMAT which is read in.

10.53	
(5E15.8)	⟵ Example. Any FORMAT, ending in column up to 60.
FF(I) I=1,NRMAX	Real part of form factor.

Reads in the imaginary part of form factor with FORMAT which is read in.

10.54	
(5E15.8)	⟵ Example. Any FORMAT, ending in column up to 60.
FFI(I) I=1,NRMAX	Imaginary part of form factor.

Microscopic inelastic excitation form factor (FFSUB6)

10.61	
IREAD(3)	type of interaction, =0; Gaussian, $V \times \exp(-\mu r^2)$ =1; Yukawa, $V \times \exp(-\mu r)/\mu r$ =2; OPEP tensor, $V \times h_2^{(1)}(i\mu r)\ S_{12}$ =3; $r^2\times$ Yukawa tensor, $V \times r^2 \exp(-\mu r)/\mu r\ S_{12}$ =4; Delta, $V \times \delta(r)$ =5; Coulomb, $z_a e^2/r$
FREAD(10)	V ; Interaction strength. (MeV)
FREAD(11)	μ ; Range parameter.
10.62	
FREAD(2)	m ; Mass of bound particle.
FREAD(3)	M ; Mass of core.
10.63	
FREAD(1)	B_{ni} ; Binding energy (must be positive).
IREAD(1)	N_i ; Number of nodes.
IREAD(4)	l_i ; Orbital angular momentum.
FREAD(12)	j_i ; Total angular momentum. for initial state.
IREAD(2)	$(zZ)_i$; Charge product.
FREAD(4)	$\triangle M_i$ (= 0, in usual case.)
FREAD(5)	$\triangle Z_i$ (= 0, in usual case.)
10.64	
FREAD(6)	r_{0i} ; Well radius parameter.
FREAD(7)	r_{ci} ; Coulomb charge radius parameter. for initial satate
FREAD(8)	a_i ; Well diffuseness parameter.
FREAD(9)	V_{soi} ; Depth of the spin-orbit well.
10.65	
FREAD(13)	B_{nf} ; Binding energy (must be positive).
IREAD(5)	N_f ; Number of nodes.
IREAD(6)	l_f ; Orbital angular momentum.
FREAD(14)	j_f ; Total angular momentum. for final state.
IREAD(10)	$(zZ)_f$; Charge product.
FREAD(15)	$\triangle M_f$ (= 0, in usual case.)
FREAD(16)	$\triangle Z_f$ (= 0, in usual case.)
10.66	
FREAD(17)	r_{0f} ; Well radius parameter.
FREAD(18)	r_{cf} ; Coulomb charge radius parameter. for final state.
FREAD(19)	a_f ; Well diffuseness parameter.
FREAD(20)	V_{sof}; Depth of the spin-orbit well.

Microscopic two-nucleon transfer form factor. (FFSUB7)

10.71		
FREAD(10)	κ ; Range parameter of t (^{3}He) or α wave function.	
FREAD(11)	η ; Range parameter of deuteron wave function.	
10.72		
FREAD(2)	m ; Mass of bound particle.	
FREAD(3)	M ; Mass of core.	
10.73		
FREAD(1)	B_{n1} ; Binding energy (must be positive).	
IREAD(1)	N_1 ; Number of nodes.	
IREAD(4)	l_1 ; Orbital angular momentum.	
FREAD(12)	j_1 ; Total angular momentum.	for particle 1.
IREAD(2)	$(zZ)_1$; Charge product.	
FREAD(4)	$\triangle M_1$ (= 0, in usual case.)	
FREAD(5)	$\triangle Z_1$ (= 0, in usual case.)	
10.74		
FREAD(6)	r_{01} ; Well radius parameter.	
FREAD(7)	r_{c1} ; Coulomb charge radius parameter.	for particle 1.
FREAD(8)	a_1 ; Well diffuseness parameter.	
FREAD(9)	V_{so1}; Depth of the spin-orbit well.	
10.75		
FREAD(13)	B_{n2} ; Binding energy (must be positive).	
IREAD(5)	N_2 ; Number of nodes.	
IREAD(6)	l_2 ; Orbital angular momentum.	
FREAD(14)	j_2 ; Total angular momentum.	for particle 2.
IREAD(10)	$(zZ)_2$; Charge product.	
FREAD(15)	$\triangle M_2$ (= 0, in usual case.)	
FREAD(16)	$\triangle Z_2$ (= 0, in usual case.)	
10.76		
FREAD(17)	r_{02} ; Well radius parameter.	
FREAD(18)	r_{c2} ; Coulomb charge radius parameter.	for particle 2.
FREAD(19)	a_2 ; Well diffuseness parameter.	
FREAD(20)	V_{so2}; Depth of the spin-orbit well.	

A blank card will terminate to read the form factor data.

blank card

1.3 Subroutine MIX

This routine calculates the differential cross section and polarization observables after coherently summing the scattering amplitude over the various processes.

$$B^{mm_bm_a}(\theta) = \sum_N a_N \beta_N^{mm_bm_a}(\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{10}{16\pi^2} \frac{1}{E_{CM_a} E_{CM_b}} \frac{k_b}{k_a} \frac{2I_B + 1}{(2s_a + 1)(2I_A + 1)} \sum_{mm_am_b} |B^{mm_bm_a}(\theta)|^2$$

The scattering amplitudes $\beta^{mm_bm_a}(\theta)$ can be stored on the permanent file. This procedure is controlled by the control integer KTOUT(1) (the first data on the "title card"). Each scattering amplitude is stored on I/O unit 8 with the identification number (=NUMRUN(5)). The starting number of NUMRUN(5) equals 1 or INS (in the case INS $\neq$ 0) and is increased with the increment one as the run goes in succession.

1.3.1 Input data for the subroutine MIX

`FORMAT(10I1,·····)`

9	(title card)

`FORMAT(3F10.5)`

	(MIX card)
N	ID-number of the scattering amplitude. If N is a negative number, the run goes immediately to the read-statement of the "title card" for the next run of TWOSTP.
AR	Real part of the factor a_N
AI	Imaginary part of the a_N

A blank card terminates the summation of the scattering amplitudes and calculates the cross section.

blank card

Then the run goes back to the read-statement of the "MIX card" for the next run.

If one wants to append the scattering amplitude on the permanent file, which already exist, one should read the final scattering amplitude on that file by subroutine MIX and after that go back to input routine of the TWOSTP.

Sample input data for this case.

9 (title card)

N. 1.

blank card

−1.

5· · · · · · INS · · · (title card)

The data of the run of TWOSTP

The INS must be N+1.

2 General description of TWOSTP

The computer program **TWOSTP** calculates the scattering differential cross section for general form of the distorted wave Born approximation up to second order. Namely, one-step and two-step processes of the reaction can be calculated. The incoming and outgoing wave may be in any combination of spin 0, spin 1/2 or spin 1 particles. The calculations are performed in a zero-range form factor between the coordinates of prior and post channel waves.

The angular momentum algebra used in this program is almost close to that of Satchler [1].

2.1 Unit system in the program

Length	;	fermi (10^{-13} cm)
Mass	;	proton mass unit
Energy	;	MeV
Cross Section	;	mb/sr.
Angle	;	degree (in the center of mass coordinate system)

2.2 Notations

final	intermediate	initial	Channel.
m_f	m_m	m_i	Mass of projectile.
$\frac{m_i}{m_f}\triangle r_i$	$\frac{m_m}{m_f}\triangle r_i$	$\triangle r_i$	Integration step size.
			Transferred angular momentum.
$l_f \quad l_2$	$l_m \quad l_1$	l_i	
$s_f \quad s_2$	$s_m \quad s_1$	s_i	Angular momentum of partial wave
$j_f \quad j_2$	$j_m \quad j_1$	j_i	
2	1		
$\sigma_f \quad \sigma'_f$	$\sigma'_m \quad \sigma_m$	$\sigma'_i \quad \sigma_i$	z-component of projectile spin.
I_f	I_m	I_i	Spin of target.
M_f	M_m	M_i	z-component of target spin.
	$l \quad s \quad j$		Total transferred angular momentum.

2.3 DWBA up to Second Order

The differential cross section is defined in terms of the transition amplitude T,

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2}\frac{k_f}{k_i}\frac{1}{(2I_i+1)(2s_i+1)}\sum_{M_i M_f \sigma_i \sigma_f}|T|^2$$

The transition amplitude of the distorted wave up to second-order Born approximation has the form

$$T = T^{one} + \sum_m T^{two}_m$$

where the summation is taken over intermediate channels.

$$\begin{aligned} T^{one} &= \langle \mathbf{k}_f; I_f M_f, s_f \sigma_f | V | \mathbf{k}_i; I_i M_i, s_i \sigma_i \rangle \\ &= J \sum_{\sigma'_i \sigma'_f} \int d\mathbf{r}_i \int d\mathbf{r}_f \chi^{(-)*}_{\sigma'_f \sigma_f}(\mathbf{k}_f, \mathbf{r}_f) \langle I_f M_f s_f \sigma'_f | V | I_i M_i s_i \sigma'_i \rangle \chi^{(+)}_{\sigma'_i \sigma_i}(\mathbf{k}_i, \mathbf{r}_i) \\ &= \sum_{lsj} \sqrt{2j+1} (I_i j M_i M_f - M_i | I_f M_f) A^{I_f I_i}_{lsj} \beta^{m_f \sigma_f \sigma_i}_{lsj}(\mathbf{k}_f, \mathbf{k}_i), \end{aligned}$$

$$\begin{aligned} T^{two} &= \langle \mathbf{k}_f; I_f M_f, s_f \sigma_f | V_2 G^{(+)}_m V_1 | \mathbf{k}_i; I_i M_i, s_i \sigma_i \rangle \\ &= J_1 J_2 \sum_{M_m \sigma'_i \sigma_m \sigma'_m \sigma'_f} \int d\mathbf{r}_i \int d\mathbf{r}_m \int d\mathbf{r}'_m \int d\mathbf{r}_f \chi^{(-)*}_{\sigma'_f \sigma_f}(\mathbf{k}_f, \mathbf{r}_f) \\ &\quad \times \langle I_f M_f s_f \sigma'_f | V_2 | I_m M_m s_m \sigma'_m \rangle G^{(+)}_{\sigma'_m \sigma_m}(\mathbf{r}'_m, \mathbf{r}_m) \langle I_m M_m s_m \sigma_m | V_1 | I_i M_i s_i \sigma'_i \rangle \\ &\quad \times \chi^{(+)}_{\sigma'_i \sigma_i}(\mathbf{k}_i, \mathbf{r}_i) \\ &= \sum_j \sqrt{2j+1} (I_i j M_i M_f - M_i | I_f M_f) \sum_{\substack{l_1 s_1 j_1 \\ l_2 s_2 j_2}} A^{I_m I_i}_{l_1 s_1 j_1} A^{I_f I_m}_{l_2 s_2 j_2} \beta^{m_f \sigma_f \sigma_i}_{j, \substack{l_1 s_1 j_1 \\ l_2 s_2 j_2}}(\mathbf{k}_f, \mathbf{k}_i), \end{aligned}$$

where $\chi^{(+)}_i$ and $\chi^{(-)}_f$ are the distorted waves, $\mathbf{r}_i$ and $\mathbf{r}_f$ are the reaction channel coordinates for the initial and final channels respectively and $G^{(+)}_m$ is the distorted wave Green function of the intermediate channel, $\mathbf{r}_m$ and $\mathbf{r}'_m$ are the reaction channel coordinates for intermediate channel and J, J_1 and J_2 are the Jacobian of the coordinate transformation of the form factor from the natural coordinates to the reaction channel coordinates. In the presence of any spin-orbit coupling the distorted wave may be written by a partial wave expansion

$$\begin{aligned} \chi^{(+)}_{m'm}(\mathbf{k}, \mathbf{r}) &= \frac{4\pi}{kr} \sum_{JLM} (LsMm | JM+m)(LsM+m-m'm' | JM+m) \\ &\quad \times i^L \chi_{LJ}(kr) Y^{M*}_L(\hat{\mathbf{k}}) Y^{M+m-m'}_L(\hat{\mathbf{r}}) \end{aligned}$$

The partial waves are solutions of the Schrödinger equation

$$[\frac{d^2}{dr^2} + k^2 - \frac{L(L+1)}{r^2} - \frac{2\mu}{\hbar^2}(U + U_c + U^J_L)] \chi_{LJ}(kr) = 0$$

with $\chi_{LJ}(0) = 0$. For large r, beyond the range of the nuclear potentials, radial waves have the form

$$\chi_{LJ}(kr) = \frac{i}{2}[H^{(-)}_L(kr) - \eta^J_L H^{(+)}_L(kr)] \exp(i\sigma_L),$$

where $H^{(+)}_L = G_L + iF_L$ is the outgoing-wave Coulomb function, and η^J_L is the reflection coefficient for the (L,J) wave. The η^J_L are obtained by matching the function and its

derivative to the solution of the Schrödinger equation. U is the central optical potential, U_c the Coulomb potential and U_L^J the spin-orbit potential for the (L,J) wave. The reciprocity relation is

$$\chi^{(+)}_{m',m}(\mathbf{k},\mathbf{r}) = (-)^{m-m'}\chi^{(+)}_{-m',-m}(-\mathbf{k},\mathbf{r}).$$

The distorted wave Green function has the form in the presence of spin-orbit coupling

$$\begin{aligned} G^{(+)}_{m'm}(\mathbf{r}',\mathbf{r}) &= \frac{2\mu}{\hbar^2}\frac{1}{kr'r}\sum_{JLM}(LsMm|JM+m)(LsM+m-m'm'|JM+m) \\ &\quad \times i^L \exp(-i\sigma_L)\chi_{LJ}(kr_<)\mathcal{H}^{(+)}_{LJ}(kr_>)Y_L^{M*}(\hat{\mathbf{k}})Y_L^{M+m-m'}(\hat{\mathbf{r}}) \end{aligned}$$

where $\mathcal{H}^{(+)}_{LJ}$ is the irregular solution which is computed by matching the function and its derivative to the outgoing wave Coulomb function $H^{(+)}_L$ at large r and integrating inward the Schrödinger equation. The total minus sign is omitted in this expression, because the attractive potentials have positive value strength in definition of this program. In the case that the kinetic energy of the propagating particle in the intermediate channel is negative, the wave number is imaginary and the Coulomb function is replaced by the corresponding Coulomb function.

The remaining factors in the transition amplitudes are the matrix elements of the interaction causing the reaction, taken between the internal states of colliding pairs in each step of the reaction processes

$$\langle I_B M_B, s_b\sigma_b|V|I_A M_A, s_a\sigma_a\rangle = \int \Psi^{M_B*}_{I_B}\Psi^{\sigma_b*}_{s_b}V\Psi^{M_A}_{I_A}\Psi^{\sigma_a}_{s_a}d\boldsymbol{\xi}$$

where $\boldsymbol{\xi}$ represents all the coordinates independent of $\mathbf{r}_a$ and $\mathbf{r}_b$. This factor contains all the information on nuclear structure, angular momentum selection rules and even the type of reaction being considered (whether stripping, knock-on or inelastic scattering etc.). This matrix element can be expanded into terms which correspond to the transfer to the nucleus of a definite angular momenta $\mathbf{l}$, $\mathbf{s}$ and $\mathbf{j}$ ($\mathbf{l}+\mathbf{s}=\mathbf{j}$).

$$\begin{aligned} &J\langle I_B M_B, s_b\sigma_b|V|I_A M_A, s_a\sigma_a\rangle \\ &\quad = \sum_{lsj} i^{-l}(-)^{s_b-\sigma_b}(I_A j M_A, M_B - M_A|I_B M_B)(s_a s_b\sigma_a, -\sigma_b|s\sigma_a-\sigma_b) \\ &\quad \times(lsm, \sigma_a-\sigma_b|jM_B-M_A)A^{I_B I_A}_{lsj}f_{lsj}(\mathbf{r}_a,\mathbf{r}_b) \end{aligned}$$

where $m = M_B - M_A + \sigma_b - \sigma_a$ and $A^{I_B I_A}_{lsj}$ is the spectroscopic amplitude.

In this program the so-called "zero-range" approximation is introduced for simplicity of numerical calculations. The form factor can then be written

$$f^{(zero)}_{lsj}(\mathbf{r}_a,\mathbf{r}_b) = F_{lsj}(r_a)Y_l^{m*}(\hat{\mathbf{r}}_a)\delta(\mathbf{r}_b - \frac{M_A}{M_B}\mathbf{r}_a).$$

The one-step and two-step transition amplitudes, by using the zero-range approximation for the form factor, may be written as follows if the z-axis is chosen along $\mathbf{k}_i$ and the y-axis along $\mathbf{k}_i \times \mathbf{k}_f$.

One-Step Process

$$
\begin{aligned}
\beta_{lsj}^{m_f\sigma_f\sigma_i}(\theta) &= (-)^{m_f+j+l+s_f-s_i}\beta_{lsj}^{-m_f,-\sigma_f,-\sigma_i}(\theta) \\
&= \sum_{\substack{l_ij_i \\ l_fj_f}} i^{l_i-l_f-l}\sqrt{\frac{(l_f-m_f)!}{(l_f+m_f)!}}P_{l_f}^{m_f}(\theta) \\
&\quad \times\sqrt{4\pi(2l_f+1)^2(2j_f+1)(2s+1)(2l+1)(2l_i+1)}\begin{Bmatrix} l_f & s_f & j_f \\ l & s & j \\ l_i & s_i & j_i \end{Bmatrix} \\
&\quad \times(l_fs_f-m_f,\sigma_f|j_f-m_f+\sigma_f)(l_is_i0\sigma_i|j_i\sigma_i) \\
&\quad \times(l_fl00|l_i0)(j_fj-m_f+\sigma_f,M_f-M_i|j_i\sigma_i) \\
&\quad \times\int\chi_{l_fj_f}(k_f,\frac{m_i}{m_f}r)F_{lsj}(\Box r)\chi_{l_ij_i}(k_i,r)dr
\end{aligned}
$$

where "□" depends on the type of reaction process.

Two-Step Process

$$
\begin{aligned}
\beta_{j,\substack{l_1s_1j_1\\l_2s_2j_2}}^{m_f\sigma_f\sigma_i}(\theta) &= (-)^{m_f+j+l_1+l_2+s_f-s_i}\beta_{j,\substack{l_1s_1j_1\\l_2s_2j_2}}^{-m_f,-\sigma_f,-\sigma_i}(\theta) \\
&= \sum_{\substack{l_ij_i\\l_mj_m\\l_fj_f}} i^{l_i-l_f-l_1-l_2}\sqrt{\frac{(l_f-m_f)!}{(l_f+m_f)!}}P_{l_f}^{m_f}(\theta) \\
&\quad \times(-)^{j_1+j_2-j} \\
&\quad \times\sqrt{(2j+1)(2I_m+1)(2j_f+1)(2l_f+1)^2(2s_2+1)(2l_2+1)(2j_2+1)} \\
&\quad \times\sqrt{(2l_m+1)(2j_m+1)^2(2s_1+1)(2l_1+1)(2j_1+1)(2l_i+1)} \\
&\quad \times\begin{Bmatrix} l_f & s_f & j_f \\ l_2 & s_2 & j_2 \\ l_m & s_m & j_m \end{Bmatrix}\begin{Bmatrix} l_m & s_m & j_m \\ l_1 & s_1 & j_1 \\ l_i & s_i & j_i \end{Bmatrix} \\
&\quad \times W(j_ij_1j_fj_2;j_mj)W(I_ij_1I_fj_2;I_mj) \\
&\quad \times(l_fs_f-m_f,\sigma_f|j_f-m_f+\sigma_f)(l_is_i0\sigma_i|j_i\sigma_i) \\
&\quad \times(l_fl_200|l_m0)(l_ml_100|l_i0)(j_fj-m_f+\sigma_f,M_f-M_i|j_i\sigma_i) \\
&\quad \times\int\int\chi_{l_fj_f}(k_f,\frac{m_i}{m_f}r')F_{l_2s_2j_2}(\Box r')G^{(+)}(\frac{m_i}{m}r',\frac{m_i}{m}r)F_{l_2s_2j_2}(\Box r) \\
&\quad \times\chi_{l_ij_i}(k_i,r)dr'\,dr
\end{aligned}
$$

2.4 Cross sections and polarization

The differential cross section for unpolarized projectiles and target nuclei is given by

$$
\frac{d\sigma}{d\Omega} = \frac{10}{16\pi^2}\,\frac{1}{E_{CM_i}E_{CM_f}}\,\frac{k_f}{k_i}\frac{2I_f+1}{(2I_i+1)(2s_i+1)}\sum_{j,m_f\sigma_f\sigma_i}|B_j^{m_f\sigma_f\sigma_i}(\theta)|^2
$$

$$B_j^{m_f\sigma_f\sigma_i}(\theta) = \sum_{ls} A_{lsj}^{I_f I_i} \beta_{lsj}^{m_f\sigma_f\sigma_i}(\theta) + \sum_{I_m, \substack{l_1 s_1 j_1 \\ l_2 s_2 j_2}} A_{l_1 s_1 j_1}^{I_m I_i} A_{l_2 s_2 j_2}^{I_f I_m} \beta_{j, \substack{l_1 s_1 j_1 \\ l_2 s_2 j_2}}^{m_f\sigma_f\sigma_i}(\theta)$$

where E_{CM}'s are the energy of the relative motion in the center of mass coordinates of the initial and final channels.

The vector polarization $\mathbf{P}_f$ of the emitted particle is defined as the expectation value $\langle s_f \rangle / s_f$. Only the component of $\mathbf{P}_f$ perpendicular to the reaction plane (this is along $\mathbf{k}_i \times \mathbf{k}_f$) is nonvanishing, if the incident beam is unpolarized.

$$P_f(\theta) = \frac{\sum_{j,m_f\sigma_f\sigma_i} \sqrt{(s_f - \sigma_f)(s_f + \sigma_f + 1)} \Im m[B_j^{m_f\sigma_f\sigma_i}(\theta) B_j^{m_f+1,\sigma_f+1,\sigma_i *}(\theta)]}{s_f \sum_{j,m_f\sigma_f\sigma_i} |B_j^{m_f\sigma_f\sigma_i}(\theta)|^2}$$

The vector polarization $P_i = \langle s_i \rangle / s_i$ can be written

$$P_i(\theta) = \frac{\sum_{j,m_f\sigma_f\sigma_i} \sqrt{(s_i - \sigma_i)(s_i + \sigma_i + 1)} \Im m[B_j^{m_f\sigma_f\sigma_i}(\theta) B_j^{m_f-1,\sigma_f,\sigma_i-1 *}(\theta)]}{s_i \sum_{j,m_f\sigma_f\sigma_i} |B_j^{m_f\sigma_f\sigma_i}(\theta)|^2}$$

2.5 Models of the reaction process

2.5.1 Inelastic scattering

If we neglect the exchange of the scattered particle with one in the nucleus and assume that the interaction is local, then the "zero-range" condition $\mathbf{r}_a = \mathbf{r}_b$ is satisfied automatically.

The interaction may be expanded in multipoles, each term being a scalar product

$$V(\mathbf{r}, \boldsymbol{\sigma}, \xi) = \sum_{lsj,\mu} (-)^{j-\mu} V_{lsj,\mu}(r, \xi) T_{lsj,-\mu}(\hat{\mathbf{r}}, \boldsymbol{\sigma})$$

$$T_{lsj,\mu}(\hat{\mathbf{r}}, \boldsymbol{\sigma}) = \sum_m (lsm, \mu - m | j\mu) i^l Y_l^m(\hat{\mathbf{r}}) \mathbf{s}_{s,\mu-m}(\sigma)$$

Hereafter we use the Wigner-Eckart theorem of the form

$$(j_2 m_2 | V_{jm} | j_1 m_1) = \frac{1}{\sqrt{2j_2 + 1}} (j_1 j m_1 \mu | j_2 m_2)(j_2 \| V_j \| j_1).$$

$$A_{lsj} F_{lsj}(r) = (-)^s \frac{1}{\sqrt{(2I_B + 1)(2s + 1)}} (I_B \| V_{lsj} \| I_A)(s_a \| \boldsymbol{\sigma}^{(s)} \| s_a)$$

where

$$s = 0; \qquad \boldsymbol{\sigma}^{(0)} = 1, \qquad (s_a \| \boldsymbol{\sigma}^{(0)} \| s_a) = \sqrt{2s_a + 1}$$

$$s = 1; \qquad \boldsymbol{\sigma}^{(1)} = \boldsymbol{\sigma}, \qquad (s_a \| \boldsymbol{\sigma}^{(1)} \| s_a) = \sqrt{s_a(s_a + 1)(2s_a + 1)}$$

In the case $s = 0$ (spin-independent interaction),

$$\begin{aligned} T_{l0l,m}(\hat{r}) &= i^l Y_l^m(\hat{\mathbf{r}}) \\ V(\mathbf{r},\xi) &= \sum_{lm} V_{lm}(r,\xi)[i^l Y_l^m(\hat{\mathbf{r}})]^* \\ A_l F_l(\mathbf{r}) &= \sqrt{\frac{2s_a+1}{2I_B+1}}(I_B\|V_l(r,\xi)\|I_A) \end{aligned}$$

(The factor $\sqrt{2s_a+1}$ is already taken into account in the program.)

2.5.1.1 Collective rotational model

We assume that the potential is non-spherical and it only depends on the distance from the surface and then allow this surface to have the following non-spherical form

$$U = U(r - R(\theta', \phi')),$$

$$R(\theta',\phi') = R_0[1 - \sum_{kq} \frac{|\alpha_{kq}|^2}{4\pi} + \sum_{kq} \alpha_{kq} Y_k^q(\theta',\phi')] = R_0 + \delta R.$$

where the polar angle (θ', ϕ') is referred to the body-fixed principal axes of the nucleus. A Taylor-series expansion about $R = R_0$ yields

$$\begin{aligned} U(r-R) &= U(r-R_0) - \delta R \frac{d}{dr}U(r-R_0) + \frac{1}{2}(\delta R)^2 \frac{d^2}{dr^2}U(r-R_0) - \cdots \\ &= V^{(0)} + V^{(1)} + V^{(2)} + \cdots \end{aligned}$$

where

$$\delta R = R_0[\sum_{kq} \alpha_{kq} Y_k^q(\theta',\phi') - \sum_{kq} \frac{|\alpha_{kq}|^2}{4\pi}]$$

Other choice of the deformation is that the equi-potential surface is parallel with the nuclear surface [2].

$$R(\theta',\phi') = R_0[1 - \sum_{kq} \frac{|\alpha_{kq}|^2}{4\pi} + \sum_{kq} \alpha_{kq} Y_k^q(\theta',\phi')] = R_0 + \delta R.$$

$$\begin{aligned} r &= r_0[1 + \sum_{kq} \alpha_{kq} Y_k^q(\theta',\phi') - \sum_{kq} \frac{|\alpha_{kq}|^2}{4\pi}] \\ U &= U(r_0) = U(r[1 + \sum_{kq} \frac{|\alpha_{kq}|^2}{4\pi} - \sum_{kq} \alpha_{kq} Y_k^q(\theta',\phi')]) \\ &= U(r) - \sum_{kq} \alpha_{kq} Y_k^q(\theta',\phi') r \frac{d}{dr}U \\ &\quad + [\sum_{kq} \alpha_{kq} Y_k^q(\theta',\phi')]^2 [r\frac{d}{dr}U + \frac{1}{2}r^2\frac{d^2}{dr^2}U] + \cdots, \\ &= U(r) - \delta R \frac{r}{R_0}\frac{d}{dr}U + \frac{1}{2}(\delta R)^2[2\frac{r}{R_0^2}\frac{d}{dr}U + \frac{r^2}{R_0^2}\frac{d^2}{dr^2}U] + \cdots, \end{aligned}$$

We must rotate into a space-fixed coordinate system, using

$$Y_k^q(\theta',\phi') = \sum_{q'} Y_k^{q'}(\theta,\phi) D^k_{q',q}(R^{-1})$$

where R is the rotation taking the body-fixed axes into coincidence with the space-fixed axes.

$$V^{(1)}_{lm} = \int V^{(1)}[i^l Y_l^m(\theta,\phi)]d\Omega = -i^l \sum_q D^l_{q,m}(R)\alpha^*_{kq} R_0 \frac{d}{dr} U(r-R_0)$$

where the following relations were used.

$$\alpha^*_{k,q} = (-)^q \alpha_{k,-q}, \qquad D^k_{q',q}(R^{-1}) = D^{k*}_{q,q'}(R) = (-)^{q-q'} D^k_{-q,-q'}(R).$$

$$\begin{aligned} V^{(2)}_{lm} &= \frac{i^l}{2}\sum_\mu D^l_{\mu,m}(R) \sum_{kk'q} [\alpha_{kq}\alpha_{k',\mu-q}]^* \sqrt{\frac{(2k+1)(2k'+1)}{4\pi(2l+1)}} (kk'q,\mu-q|l\mu) \\ &\quad \times (kk'00|l0) R_0^2 \frac{d^2}{dr^2} U(r-R_0) \end{aligned}$$

These have a common form

$$V^{(i)}_{l,m} = \sum_\mu \mathcal{V}^{(i)}_{l,\mu} D^l_{\mu,m}(R)$$

The wave functions for an asymmetric rotor are written

$$\begin{aligned} \Phi_{\alpha IM} &= \sum_K A^\alpha_K \phi_{KIM} \\ \phi_{KIM} &= \frac{1}{4\pi}\sqrt{\frac{2I+1}{1+\delta_{K0}}}\{D^I_{K,M}(R)\chi_K + (-)^{I+K} D^I_{-K,M}(R)\chi_{-K}\} \end{aligned}$$

Then the reduced matrix element is written as

$$\begin{aligned} \langle \beta I_B \| V \| \alpha I_A \rangle &= \sum_{K_A K_B} A^\alpha_{K_A} A^\beta_{K_B} \langle K_B I_B \| V \| K_A I_A \rangle \\ \langle K_B I_B \| V \| K_A I_A \rangle &= \sqrt{\frac{2I_A+1}{(1+\delta_{K_A 0})(1+\delta_{K_B 0})}} \\ &\quad \times \left[(I_A l K_A, K_B - K_A | I_B K_B) \mathcal{V}_{l,K_B-K_A} + (-)^{I_A} (I_A l - K_A, K_B + K_A | I_B K_B) \mathcal{V}_{l,K_B+K_A} \right] \end{aligned}$$

For even target nuclei we have $I_A = K_A = 0$ and $I_B = l$,

$$\langle \beta l \| V^{(i)} \| \alpha 0 \rangle = \sum_{K_B} A^\beta_{K_B} \sqrt{\frac{2}{1+\delta_{K_B 0}}} \mathcal{V}^{(i)}_{l,K_B}$$

In the axially aymmetric case, with $K_B = 0$ only,

$$\langle \beta l \| V^{(i)} \| \alpha 0 \rangle = \mathcal{V}^{(i)}_{l,0}$$

2.5.1.2 Collective vibrational model

The model adopted here is the same as for rotations, namely, a nonspherical potential well oscillates about a spherical mean. This is achieved by treating the deformation parameters α_{kq} as dynamical variables

$$\alpha_{kq} = \sqrt{\frac{\hbar\omega_k}{2C_k}}\{b_{kq} + (-)^q b^*_{k,-q}\} = (-)^q \alpha^*_{k,-q}$$

where the b^*_{kq} (b_{kq}) is the usual boson creation (annihilation) operators for a 2^k-pole oscillation with angular momentum k and z-component q. The energy of a phonon is $\hbar\omega_k$, and C_k is the restoring-force parameter.

$$\begin{aligned} V^{(1)}_{lm} &= -i^l \alpha^*_{lm} R_0 \frac{d}{dr} V^{(0)} \\ V^{(2)}_{lm} &= \frac{i^l}{2} \sum_{kk'q} [\alpha_{kq}\alpha_{k',m-q}]^* \sqrt{\frac{(2k+1)(2k'+1)}{4\pi(2l+1)}} (kk'q, m-q|lm) \\ &\quad \times (kk'00|l0) R_0^2 \frac{d^2}{dr^2} V^{(0)} \end{aligned}$$

In the case of even target nuclei, which has zero spin, the wave function of various vibrational states are presented as follows

(i) no-phonon (ground) state $|0\rangle$

(ii) one-phonon state $\qquad |1; IM\rangle = b^*_{IM}|0\rangle$

(iii) two-phonon state

$$|2; IM\rangle = \frac{1}{\sqrt{1+\delta_{kk'}}} \sum_q (kk'q, M-q|IM) b^*_{kq} b^*_{k',M-q}|0\rangle$$

2.5.1.3 Microscopic inelastic excitation

The form factor of a single particle excitation by a central or tensor force between a nucleon in the projectile and a nucleon in the target nucleus is given

$$F_{c',c} = \langle I_B M_B, s_b m_b | \sum_{ij} V_{ij} | I_A M_A, s_a m_a \rangle$$

where $|I_\alpha M_\alpha, s_\alpha m_\alpha\rangle$'s are the initial wave function of colliding pair and the summation i runs over the nucleon in the projectile and j runs over the nucleon in the target nucleus.

The interaction has the form

$$\begin{aligned} V_{ij} &= \sum_{st} V^{C(s,t)}_{ij}(|\mathbf{r}_i - \mathbf{r}_j|)(\boldsymbol{\sigma}^{(s)}_i \cdot \boldsymbol{\sigma}^{(s)}_j)(\boldsymbol{\tau}^{(t)}_i \cdot \boldsymbol{\tau}^{(t)}_j); \text{ central force} \\ \text{or} \quad &= \sum_t V^{T(t)}_{ij}(|\mathbf{r}_i - \mathbf{r}_j|)(\boldsymbol{\tau}^{(t)}_i \cdot \boldsymbol{\tau}^{(t)}_j) S_{ij} \qquad ; \text{tensor force} \end{aligned}$$

where s and t are the spin and isospin transfer to the nucleus. The $\boldsymbol{\sigma}^{(s)}$ and $\boldsymbol{\tau}^{(t)}$ are the spin and isospin operators,

$$\boldsymbol{\sigma}^{(0)} = 1, \qquad \boldsymbol{\tau}^{(0)} = 1$$
$$\boldsymbol{\sigma}^{(1)} = \boldsymbol{\sigma}, \qquad \boldsymbol{\tau}^{(1)} = \boldsymbol{\tau}$$

The S_{ij} is the tensor operator

$$S_{ij} = \frac{3(\boldsymbol{\sigma}_i \cdot \mathbf{r})(\boldsymbol{\sigma}_j \cdot \mathbf{r})}{r^2} - (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) = \sqrt{\frac{24\pi}{5}}([\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]^{(2)} \cdot Y^{(2)}(\hat{\mathbf{r}}))$$

where $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$.

The multipole expansion of the spatial part of the interaction is given for the central force

$$V_{ij}^{C(s,t)}(|\mathbf{r}_i - \mathbf{r}_j|) = \sum_{lm} v_l^{C(s,t)}(r_i, r_j) Y_l^{m*}(\hat{\mathbf{r}}_i) Y_l^m(\hat{\mathbf{r}}_j),$$

and for the tensor force

$$\begin{aligned} V_{ij}^{T(t)}(|\mathbf{r}_i - \mathbf{r}_j|) \cdot Y_\mu^{(2)} &= \sum_{l_i l_j} v_{l_i l_j}^{T(t)}(r_i, r_j) \sqrt{\frac{(2l_i+1)(2l_j+1)}{4\pi \cdot 5}} \\ &\quad \times (l_i l_j 00|20)[Y_{l_i}(\hat{\mathbf{r}}_i) \times Y_{l_j}(\hat{\mathbf{r}}_j)]_\mu^{(2)} \end{aligned}$$

The common form of the multipole expansion of the central (k=0) and tensor (k=2) force interaction is given

$$\left.\begin{matrix}(\boldsymbol{\sigma}_i^{(s)} \cdot \boldsymbol{\sigma}_j^{(s)})(\boldsymbol{\tau}_i^{(t)} \cdot \boldsymbol{\tau}_j^{(t)}) V(r) \\ (\boldsymbol{\tau}_i^{(t)} \cdot \boldsymbol{\tau}_j^{(t)}) S_{ij} V(r)\end{matrix}\right\} = A(k)\sqrt{2s+1} \sum_{l_i l_j, jm, \sigma\mu} i^{l_i - l_j}(-)^{s-j+\sigma+\nu}$$
$$\times \sqrt{(2l_i+1)(2l_j+1)}(l_i l_j 00|k0)\ W(l_i l_j s s; kj)\ (lsm\sigma|j\mu)$$
$$\times i^{-l_i} Y_{l_i}^{m*}(\hat{r}_i) \boldsymbol{\sigma}_\sigma^{(s)} \boldsymbol{\tau}_\nu^{(t)} T_{lsjt;\mu\nu}(r_j, \boldsymbol{\sigma}_j^{(s)}, \boldsymbol{\tau}_j^{(t)})$$

where

$$T_{lsjt;\mu\nu}(r, \boldsymbol{\sigma}^{(s)}, \boldsymbol{\tau}^{(t)}) = \sum_m (lsm, \mu - m|j\mu) i^l Y_l^m(r) \boldsymbol{\sigma}_{\mu-m}^{(s)} \boldsymbol{\tau}_\nu^{(t)}$$

and

$$A(0) = 1, \qquad A(2) = -\sqrt{2}.$$

The matrix element of the projectile system is given

$$\begin{aligned} &\langle s_b m_b; t_b \nu_b | \boldsymbol{\sigma}_{-\sigma}^{(s)} \boldsymbol{\tau}_{-\nu}^{(t)} | s_a m_a; t_a \nu_a \rangle \\ &= \frac{1}{\sqrt{(2s+1)(2t+1)}} (s_b t_b \| C^{st} \| s_a t_a) \\ &\times \frac{1}{\sqrt{(2s_b+1)(2t_b+1)}} (s_a s m_a, -\sigma | s_b m_b)(t_a t \nu_a, -\nu | t_b \nu_b) \\ &\times (\frac{1}{2} \| \boldsymbol{\sigma}^{(s)} \| \frac{1}{2})(\frac{1}{2} \| \boldsymbol{\tau}^{(t)} \| \frac{1}{2}). \end{aligned}$$

where the operator C^{st} is defined as follows

$$\begin{aligned} C^{st}_{\sigma\nu} &= \sqrt{\frac{(2s+1)(2t+1)}{5}}\sum_{\sigma_1\nu_1}(\frac{1}{2}s\sigma_1\sigma|\frac{1}{2}\sigma_2)(\frac{1}{2}t\nu_1\nu|\frac{1}{2}\nu_2) \\ &\qquad \times C^{\frac{1}{2}\frac{1}{2}+}_{\sigma_2\nu_2}C^{\frac{1}{2}\frac{1}{2}}_{\sigma_1\nu_1} \\ &= \sum_{\sigma_1\nu_1}(-)^{1-\sigma_1-\nu_1}(\frac{1}{2}\frac{1}{2}\sigma_2,-\sigma_1|s\sigma)(\frac{1}{2}\frac{1}{2}\nu_2,-\nu_1|t\nu) \\ &= [C^{\frac{1}{2}\frac{1}{2}+}\times C^{\frac{1}{2}\frac{1}{2}}]^{st}_{\sigma\nu} \end{aligned}$$

We introduce the creation operator $C^{\frac{1}{2}\frac{1}{2}+}$ associated with the 0s-state in the projectile and annihilation operator $C^{\frac{1}{2}\frac{1}{2}}$ is the Hermitian conjugate of the creation operator. If we modify the annihilation operator as follows

$$\widetilde{C}^{\frac{1}{2}\frac{1}{2}}_{\sigma\nu} = (-)^{1-\sigma-\nu}C^{\frac{1}{2}\frac{1}{2}}_{-\sigma,-\nu}$$

these $C^{\frac{1}{2}\frac{1}{2}+}$ and $\widetilde{C}^{\frac{1}{2}\frac{1}{2}}$ are the components of the spherical tensor of rank 1/2 in both spin and isospin space.

The matrix element of the target nucleus is given in the n-p representation

$$\begin{aligned} &\langle I_BM_B|T_{lsjt;\mu\nu}|I_AM_A\rangle \\ &\quad = \sum_{j_1j_2}\frac{1}{\sqrt{2j+1}}(I_B\|A_j(j_2j_1;\nu_2\nu_1)\|I_A) \\ &\quad \times\frac{1}{\sqrt{2(2I_B+1)}}(I_AjM_A\mu|I_BM_B)(\frac{1}{2}t\nu_1\nu|\frac{1}{2}\nu_2)(j_2\|T_{lsj}\|j_1)(\frac{1}{2}\|\boldsymbol{\tau}^{(t)}\|\frac{1}{2}) \end{aligned}$$

where the operator of the single particle excitation has the form

$$\begin{aligned} A_{j\mu}(j_2j_1;\nu_2\nu_1) &= \sqrt{\frac{2j+1}{2j_2+1}}\sum_{m_1}(j_1jm_1\mu|j_2m_2)a^+_{j_2m_2}(\nu_2)a_{j_1m_1}(\nu_1) \\ &= \sum_{m_1}(-)^{j_1-m_1}(j_2j_1m_2,-m_1|j\mu)a^+_{j_2m_2}(\nu_2)a_{j_1m_1}(\nu_1) \\ &= (-)^{2j_1}[a^+_{j_2}(\nu_2)\times a_{j_1}(\nu_1)]^j_\nu \end{aligned}$$

Here we introduce the creation operator a^+_{jm} associated with the state $|jm\rangle$ and the annihilation operator a_{jm} that is the Hermitian conjugate of the creation operator a^+_{jm}. If we modify the annihilation operator as follows

$$\widetilde{a}_{jm} = (-)^{j-m}a_{j,-m}$$

these a^+_{jm} and $\widetilde{a}_{jm}$ are components of the spherical tensors of rank j.

The final result of the form factor is given

$$F_{c'c} = \sum_{lsj,l'j_1J_2}\frac{1}{\sqrt{(2s+1)(2t+1)}}(s_bt_b\|C^{st}\|s_at_a)$$

$$\times\frac{1}{\sqrt{2j+1}}(I_B\|A_j(j_2j_1;\nu_2\nu_1)\|I_A)$$
$$\times(-)^{s_b-m_b}(t_at\nu_a,-\nu|t_b\nu_b)(\frac{1}{2}t\nu_1\nu|\frac{1}{2}\nu_2)$$
$$\times 4\pi A(k)(2s+1)(2l'+1)\sqrt{\frac{2(2l+1)(2j+1)(2j_1+1)(2j_2+1)(2l_1+1)}{\pi(2I_B+1)}}$$
$$\times(-)^{s-j+\nu}i^{l_1+l-l_2}(ll'00|k0)\ W(ll'ss;kj)(l_1l'00|l_20)$$
$$\times\left\{\begin{array}{ccc} l_2 & \frac{1}{2} & j_2 \\ l_1 & \frac{1}{2} & j_1 \\ l' & s & j \end{array}\right\}\int\psi_{l_2j_2}(r_t)R_{ll'}(r_p,r_t)\psi_{l_1j_1}(r_t)r_t^2dr_t$$

The spectroscopic amplitude which must be input is
a_{lsj} = (the spectroscopic amplitude of the projectile system)
× (the spectroscopic amplitude of the target system).

The spectroscopic amplitude of the 0s-projectile system

$$\frac{1}{\sqrt{(2s+1)(2t+1)}}(nJT\|C^{st}\|nJ'T')$$
$$=\sum_{J"T"}(-)^{s+t}\ W(J\frac{1}{2}J'\frac{1}{2};J"s)\ W(T\frac{1}{2}T'\frac{1}{2};T"t)$$
$$\times(nJT\|C^{+}\|n-1J"T")(n-1J"T"\|\widetilde{C}\|nJ'T')$$
$$=\sum_{J"T"}(-)^{s+t}\ W(J\frac{1}{2}J'\frac{1}{2};J"s)\ W(T\frac{1}{2}T'\frac{1}{2};T"t)$$
$$\times(-)^{J'+\frac{1}{2}-J"+T'+\frac{1}{2}-T"}n\sqrt{(2J+1)(2J'+1)(2T+1)(2T'+1)}$$
$$\times\langle nJT\{|n-1J"T"\rangle\langle n-1J"T"|\}nJ'T'\rangle$$

	n=1	n=2	n=3	n=4
reaction	(p,p′),(p,n) (n,n′),(n,p)	(d,d′)	(h,h′),(t,t′) (h,t),(t,h)	(α,α')
(s=0,t=0)	1	$\sqrt{3}$	3	2
(s=1,t=0)	1	$\sqrt{2}$	1	0
(s=0,t=1)	1	0	1	0
(s=1,t=1)	1	0	-1	0

The spectroscopic amplitude of the target system in the n-p representation

$$\frac{1}{\sqrt{2j+1}}(j_2^{n_2+1}(J_2')j_1^{n_1-1}(J_1');J'\|A_j(j_2j_1;\sigma_2\sigma_1)\|j_2^{n_2}(J_2)j_1^{n_1}(J_1);J)$$
$$=-\sqrt{(2J+1)(2J'+1)}\left\{\begin{array}{ccc} J_2' & J_1' & J' \\ J_2 & J_1 & J \\ j_2 & j_1 & j \end{array}\right\}(j_2^{n_2+1};J_2'\|a^+_{j_2\sigma_2}\|j_2^{n_2};J_2)(j_1^{n_1-1};J_1'\|\widetilde{a}_{j_1\sigma_1}\|j_1^{n_1};J_1)$$

$$= (-)^{n_1+n_2+J_1+J_1-j_1-J_1}\sqrt{n_1(n_2+1)(2j+1)(2J_1+1)(2J_2'+1)(2J+1)(2J'+1)}$$
$$\times \left\{ \begin{array}{ccc} J_2' & J_1' & J' \\ J_2 & J_1 & J \\ j_2 & j_1 & j \end{array} \right\} \langle j_2^{n_2}(J_2), j_2|\}j_2^{n_2+1}J_2'\rangle\langle j_1^{n_1-1}(J_1'), j_1|\}j_1^{n_1}J_1\rangle$$

The phase convention of the shell model amplitude in this program

(1) The coupling order of the $\mathbf{l}$ and $\mathbf{s}$ is $\mathbf{l}+\mathbf{s}=\mathbf{j}$.

(2) The radial wave function of a single particle is positive near the origin.

(3) The spherical harmonics has the time reversal phase i^l, then $i^l Y_l^m$.

Mutipole expansions of the central and tensor forces are given as follows

Gaussian interaction

$$\exp(-\mu r^2) = \exp[-\mu(r_1^2+r_2^2)]4\pi\sum_{lm} B_l(2\mu r_1 r_2)Y_l^{m*}(\hat{r}_1)Y_l^m(\hat{r}_2)$$

Yukawa interaction

$$\frac{\exp(-\mu r)}{\mu r} = 4\pi\sum_{lm} B_l(\mu r_<)H_l(\mu r_>)Y_l^{m*}(\hat{r}_1)Y_l^m(\hat{r}_2)$$

Delta interaction

$$\delta(r) = \frac{\delta(r_1-r_2)}{r_1 r_2}\sum_{lm} Y_l^{m*}(\hat{r}_1)Y_l^m(\hat{r}_2)$$

Coulomb interaction

$$\frac{1}{r} = 4\pi\sum_{lm}\frac{r_<^l}{r_>^{l+1}}\frac{1}{2l+1}Y_l^{m*}(\hat{r}_1)Y_l^m(\hat{r}_2)$$

OPEP tensor interaction

$$V(r) = h_2^{(1)}(i\mu r) = \frac{\exp(-\mu r)}{\mu r}(1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2})$$

$$h_2^{(1)}(i\mu r)Y_2^m(r) = -\sqrt{4\pi}\sum_{l_1 l_2} i^{l_1-l_2}\sqrt{\frac{(2l_1+1)(2l_2+1)}{5}}(l_1 l_2 00|20)$$
$$\times[Y_{l_1}(\hat{r}_1)\times Y_{l_2}(\hat{r}_2)]_m^{(2)}\left\{ \begin{array}{ll} B_{l_1}(\mu r_1)H_{l_2}(\mu r_2) & r_1<r_2 \\ -\dfrac{\delta(r_1-r_2)}{\mu^3 r_1 r_2} & r_1=r_2 \\ H_{l_1}(\mu r_1)B_{l_2}(\mu r_2) & r_1>r_2 \end{array} \right.$$

$r^2\times$ Yukawa tensor interaction

$$r^2 \times \frac{\exp(-\mu r)}{\mu r} Y_2^m(r) = \sqrt{4\pi} \sum_{l_1 l_2 l'} \sqrt{\frac{(2l_1+1)(2l_2+1)}{5}} (l_1 l_2 00|20)[Y_{l_1}(\hat{r}_1) \times Y_{l_2}(\hat{r}_2)]^{(2)}_m$$
$$\times [r_1^2 B_{l_2}(\mu r_<) H_{l_2}(\mu r_>) - a r_1 r_2 B_{l'}(\mu r_<) H_{l'}(\mu r_>) + r_2^2 B_{l_1}(\mu r_<) H_{l_1}(\mu r_>)]$$
$$a = \begin{cases} 2 & l_2 = l_1 \pm 2 \text{ and } l' = l_1 \pm 1 \\ \frac{2l_1+3}{2l_1+1} & l_2 = l_1 \text{ and } l' = l_1 - 1 \\ \frac{2l_1-1}{2l_1+1} & l_2 = l_1 \text{ and } l' = l_1 + 1 \end{cases}$$

where

$$B_l(\rho) = (-i)^l j_l(i\rho) \text{ and } H_l(\rho) = -i^l h_l^{(1)}(i\rho)$$

and j_l, $h_l^{(1)}$ are the spherical Bessel and the first kind spherical Hankel functions.

2.5.2 Transfer Reaction

2.5.2.1 Stripping Reaction

The internal wave functions of the colliding pairs are given

$$\Phi_{I_B}^{M_B}(\mathbf{r}, \xi, \xi_A) = \sum_{j\mu I_{A'} M_{A'}} (I_{A'} j M_{A'} \mu | I_B M_B) \Phi_{I_{A'}}^{M_{A'}}(\xi_A) \Omega_j^{BA',\mu}(\mathbf{r}, \xi)$$
$$\Omega_j^{BA,\mu}(\mathbf{r}, \xi) = \sum_{lsm} (lsm, \mu - m | j\mu) \mathcal{J}(B; Alj) u_{lj}(r) i^l Y_l^m(\hat{r}) \phi_s^{\mu-m}(\xi)$$
$$\phi_{s_a}^{m_a}(\mathbf{r}_{bx}, \xi_x, \xi_b) = \sum_{s\mu} (s_b s m_b, \mu | s_a m_a) j(a; bx) \phi_{bx}(\mathbf{r}_{bx}) \phi_s^{\mu}(\xi) \phi_{s_b}^{m_b}(\xi_b)$$

We assume that the transferred particle is the s-state in the projectile and use the zero-range approximation

$$V_{bx}(\mathbf{r}_{bx}) \phi_{bx}(\mathbf{r}_{bx}) \sim D_0 \delta(\mathbf{r}_{bx}) = D_0 J^{-1} \delta(\mathbf{r}_b - \frac{m_A}{m_B} \mathbf{r}_a)$$

where J is the Jacobian of the transformation from the "natural" variables $\mathbf{r}_{xA}$ and $\mathbf{r}_{bx}$ to the $\mathbf{r}_a$ and $\mathbf{r}_b$ used for the distorted waves

$$J = [\frac{m_a m_B}{m_x (m_a + m_A)}]^3.$$

This approximation has the physical meaning that particle b is emitted at the same point where particle a is absorbed.

Then the nuclear matrix element is given

$$J\langle I_B M_B, s_b m_b | V | I_A M_A, s_a m_a \rangle$$
$$= \sum_{lsj} i^{-l} (-)^{s_b - m_b} (I_A j M_A, M_B - M_A | I_B M_B) (s_a s_b m_a, -m_b | s m_a - m_b)$$
$$\times (lsm, m_a - m_b | j M_B - M_A) s(lsj) a(s) \sqrt{\frac{2s_a + 1}{2s+1}} D_0$$
$$\times F_{lj}(r_a) Y_l^{m*}(\hat{r}_a) \delta(\hat{r}_b - \frac{m_A}{m_B} \hat{r}_a),$$

$$s(lsj) = \sqrt{n}\mathcal{J}(B; Alj), \qquad a(s) = \sqrt{\nu}j(a; bx),$$

where s and a are the spectroscopic amplitudes of the target and projectile system, respectively. The factor $\sqrt{n}$ and $\sqrt{\nu}$ arise from antisymmetry considerations. n is the number of equivalent nucleons in the orbit lj of the heavier nucleus B and ν is the number of the same particles in the projectile a as the transfer particle.

2.5.2.2 Pick-up Reaction

The internal wave functions of the colliding pairs are given

$$\begin{aligned}
\Phi_{I_A}^{M_A}(\mathbf{r}, \xi, \xi_B) &= \sum_{j\mu I_{B'} M_{B'}} (I_{B'} j M_{B'} \mu | I_A M_A) \Phi_{I_{B'}}^{M_{B'}}(\xi_B) \Omega_j^{AB',\mu}(\mathbf{r}, \xi) \\
\Omega_j^{AB,\mu}(\mathbf{r}, \xi) &= \sum_{lsm} (lsm, \mu - m | j\mu) \mathcal{J}(A; Blj) u_{lj}(r) i^l Y_l^m(\hat{r}) \phi_s^{\mu - m}(\xi) \\
\phi_{s_b}^{m_b}(\mathbf{r}_{ax}, \xi_x, \xi_a) &= \sum_{s\mu} (s_a s m_a, \mu | s_b m_b) j(b; ax) \phi_{ax}(\mathbf{r}_{ax}) \phi_s^{\mu}(\xi) \phi_{s_a}^{m_a}(\xi_a)
\end{aligned}$$

Then the nuclear matrix element is given

$$\begin{aligned}
&J\langle I_B M_B, s_b m_b | V | I_A M_A, s_a m_a \rangle \\
&= \sum_{lsj} i^l (-)^{s_a - m_a} (I_B j M_B, M_A - M_B | I_A M_A)(s_b s_a m_b, -m_a | s m_b - m_a) \\
&\quad \times (lsm, m_b - m_a | j M_A - M_B) s(lsj) a(s) \sqrt{\frac{2s_b + 1}{2s + 1}} D_0 \\
&\quad \times F_{lj}(r_b) Y_l^m(\hat{r}_b) \delta(\hat{r}_a - \frac{m_B}{m_A} \hat{r}_b), \\
&= \sum_{lsj} (-)^{I_A + j - I_B + s_b + s - s_a} \sqrt{\frac{(2I_A + 1)(2s_b + 1)}{(2I_B + 1)(2s_a + 1)}} \\
&\quad \times i^{-l} (-)^{s_b - m_b} (I_A j M_A, M_B - M_A | I_B M_B)(s_a s_b m_a, -m_b | s m_a - m_b) \\
&\quad \times (lsm, m_a - m_b | j M_B - M_A) s(lsj) a(s) \sqrt{\frac{2s_a + 1}{2s + 1}} D_0 \\
&\quad \times F_{lj}(r_b) Y_l^m(\hat{r}_b) \delta(\hat{r}_a - \frac{m_B}{m_A} \hat{r}_b),
\end{aligned}$$

$$s(lsj) = \sqrt{n}\mathcal{J}(A; Blj), \qquad a(s) = \sqrt{\nu}j(b; ax),$$

where n is the number of equivalent nucleons in the orbit lj of the heavier nucleus A and ν is the number of the same particles in the projectile b as the transfer particle.

In this program, only $a_{lsj} = s(lsj) \times a(s)$ and D_0^2 may be input for both stripping and pick-up case. About the spectroscopic factors one can see, for example, the article by Macfarlane and French [4].

2.5.2.3 Bound state wave function

The bound state function is the solution of the Schrödinger equation,

$$
\begin{aligned}
&[-\frac{\hbar^2}{2\mu}\{\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\}-E+U(r)]u_{nlj}(r)=0\\
&U(r)=-V_0 f_0(r)+V_{so}f_{so}(r)\mathbf{l}\cdot\mathbf{s}+f_c(r)\\
&f_0(r)=\frac{1}{1+\exp[(r-R_0)/a]}, \qquad R_0=r_0\times M^{1/3}\\
&f_{so}(r)=2(\frac{\hbar}{m_\pi c})^2\frac{1}{r}\frac{d}{dr}f_0(r)\\
&f_c(r)=\begin{cases}\frac{zZe^2}{2R_c}(3-\frac{r^2}{R_c^2}) & r\le R_c\\ \frac{1}{r}zZe^2 & r\ge R_c\end{cases}
\end{aligned}
$$

In the potential depth search procedure, only the central force strength is searched and the strength of spin-orbit potential is fixed. The number of nodes is counted excluding the origin and infinity. The convention of phase is that the sign of radial wave function is always positive near the origin.

2.5.2.4 Values of D_0^2

For the (d,p) or (p,d) reaction the wave function of deuteron obeys the following Schrödinger equation

$$
\{-\frac{\hbar^2}{2\mu_d}\nabla^2+V(\mathbf{r})\}\psi_d(\mathbf{r})=\epsilon\psi_d(\mathbf{r}), \qquad \epsilon=-\frac{\hbar^2}{2\mu_d}\alpha^2=-2.22\mathrm{MeV}
$$

$$
D_0=\int V(\mathbf{r})\psi_d(\mathbf{r})d\mathbf{r}=\frac{\hbar^2}{2\mu_d}\int(\nabla^2-\alpha^2)\psi_d(\mathbf{r})d\mathbf{r}.
$$

Yukawa type

$$
\psi_d=\sqrt{\frac{\alpha}{2\pi}}\frac{\exp(-\alpha r)}{r}, \qquad D_0^2=\frac{8\pi\epsilon^2}{\alpha^3}\approx 1.018\times 10^4\mathrm{MeV}^2\mathrm{fm}^3
$$

Hulthen type

$$
\psi_d=\sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}}\frac{\exp(-\alpha r)-\exp(-\beta r)}{r}, D_0^2=\frac{8\pi\alpha^2}{\alpha^3}(\frac{\alpha+\beta}{\beta})^3\approx 1.53\times 10^4\mathrm{MeV}^2\mathrm{fm}^3
$$

where $\beta(\sim 7\alpha)$ is obtained from the n-p scattering and effective range theory.

For other reactions following values are often used.

Reaction	$D_0^2\,[10^4\mathrm{MeV}^2\mathrm{fm}^3]$
(^{3}He,d)	4.42
(t,d)	5.06
(α,d)	($24\sim 46$)
(α,t)	($24\sim 46$)

2.5.2.5 Two-nucleon transfer reaction

The microscopic two-nucleon transfer form factor is calculated by Bayman-Kallio method [5]. In the case of (t,p) reaction the internal wave functions of the colliding pairs are given

$$\Phi_{I_B}^{M_B}(\xi_A, x_1, x_2) = \sum_{M'_A}(I'_A j M'_A \mu | I_B M_B)\Phi_{I'_A}^{M'_A}(\xi_A)\Phi_{j,\mu}^{BA'}(x_1, x_2)$$

$$\Phi_{j,\mu}^{BA}(x_1, x_2) = \sum_{ls}(lsm, \mu - m|j\mu) \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2}} \mathcal{J}^{BA}(n_1 l_1 j_1, n_2 l_2 j_2; j\mu)$$

$$\times\sqrt{(2l+1)(2s+1)(2j_1+1)(2j_2+1)} \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ l & s & j \end{Bmatrix}$$

$$\times\frac{[\psi_{l_1}(\mathbf{r}_1)\times\psi_{l_2}(\mathbf{r}_2)]_l^m + (-)^{1+s+t}[\psi_{l_1}(\mathbf{r}_2)\times\psi_{l_2}(\mathbf{r}_1)]_l^m}{\sqrt{2(1+\delta_{n_1 n_2}\delta_{l_1 l_2}\delta_{j_1 j_2})}}\chi_{s,t}^{\mu-m,\nu}(\eta_1,\eta_2)\delta_{s+t,1}$$

where

$$\chi_{s,t}^{m,\nu}(\eta_1,\eta_2) = [\chi_{\frac{1}{2},\frac{1}{2}}(\eta_1)\times\chi_{\frac{1}{2},\frac{1}{2}}(\eta_2)]_{s,t}^{m,\nu}$$

We introduce relative and center of mass coorinates defined by

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \qquad \text{and} \qquad \mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2},$$

and expand into the form

$$\frac{[\psi_{l_1}(\mathbf{r}_1)\times\psi_{l_2}(\mathbf{r}_2)]_l^m + [\psi_{l_1}(\mathbf{r}_2)\times\psi_{l_2}(\mathbf{r}_1)]_l^m}{\sqrt{2(1+\delta_{n_1 n_2}\delta_{l_1 l_2}\delta_{j_1 j_2})}} = \sum_{\lambda\Lambda}\frac{f_{\lambda\Lambda}^l(r,R)}{rR}[Y_\lambda(\hat{\mathbf{r}})\times Y_\Lambda(\hat{\mathbf{R}})]_l^m$$

If we assume $\lambda = 0$, with the aid of Bayman-Kallio method

$$f_{0l}^l(r,R) = \frac{1+(-)^{l_1+l_2-l}}{2}\sqrt{\frac{(2l_1+1)(2l_2+1)}{2(2l+1)(1+\delta_{n_1 n_2}\delta_{l_1 l_2}\delta_{j_1 j_2})}}$$

$$\times rR\int_{-1}^{1} u_{l_1}(r_1)u_{l_2}(r_2)[(l_1 l_2 00|l0)d_{0,0}^{l_1}(\theta_1)d_{0,0}^{l_2}(\theta_2)$$

$$+2\sum_{m>0}(l_1 l_2 m, -m|l0)d_{m,0}^{l_1}(\theta_1)d_{-m,0}^{l_2}(\theta_2)]dx,$$

where

$$r_1 = \sqrt{R^2 + \frac{1}{4}r^2 - Rrx}, \qquad \cos(\theta_1) = \frac{R - \frac{1}{2}rx}{r_1},$$

$$r_2 = \sqrt{R^2 + \frac{1}{4}r^2 + Rrx}, \qquad \cos(\theta_2) = \frac{R + \frac{1}{2}rx}{r_2}.$$

We assume that the projectile system has the 0s-state radial wave function

$$\phi_{s_a,t_a}^{m_a,\nu_a}(\xi_b, x_1, x_2) = \sum_{m'_b\nu'_b}(s_b s m'_b, m_a - m'_b|s_a m_a)(t_b t\nu'_b, \nu_a - \nu'_b|s_a\nu_a)$$

$$\times j^{ab}(s,t)\chi_{s_b,t_b}^{m_b,\nu_b}(\eta_b)\chi_{s,t}^{m_a-m'_b,\nu_a-\nu'_b}(\eta_1,\eta_2)\phi(\mathbf{r}_b, r_1, r_2)$$

and

$$\begin{aligned}\phi(\mathbf{r}_b, r_1, r_2) &= N_3 \exp[-\kappa^2 \sum_{ij} (\mathbf{r}_i - r_j)^2] \\ &= N_3 \exp[-\kappa^2 (\frac{3}{2} r^2 + 2\rho^2)], \qquad N_3 = 3^{3/4} (\frac{2\kappa^2}{\pi})^{3/2}.\end{aligned}$$

We introduce the following zero-range approximation

$$[V_{1b}(\mathbf{r}_{1b}) + V_{2b}(\mathbf{r}_{2b})]\phi(\mathbf{r}_b, \mathbf{r}_1, \mathbf{r}_2) \approx d_0(\mathbf{r})\delta(\boldsymbol{\rho}) = J^{-1} d_0(\mathbf{r})\delta(\mathbf{r}_b - \frac{m_A}{m_B}\mathbf{r}_a)$$

where J is the Jacobian of the coordinate transformation and most simple form of $d_0(r)$ is given

$$d_0(r) = D_0 N_3 \exp(-\frac{3}{2}\kappa^2 r^2)$$

The nuclear matrix element is then given

$$\begin{aligned}&J\langle I_B M_B s_b m_b | V | I_A M_A s_a m_a \rangle \\ &= \sum_{lsj} (-)^{s_b - m_b} (I_A j M_A, M_B - M_A | I_B M_B)(s_a s_b m_a, -m_b | s m_a - m_b) \\ &\quad \times (lsm, m_a - m_b | j M_A - M_B) i^{-l} Y_l^{m*}(\hat{r}_a) \\ &\quad \times \sum_{\substack{n_1 l_1 j_1 \\ n_2 l_2 j_2}} \mathcal{J}^{BA}(n_1 l_1 j_1, n_2 l_2 j_2; st) i^{l - l_1 - l_2} \\ &\quad \times \sqrt{(2l+1)(2s+1)(2j_1+1)(2j_2+1)} \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ l & s & j \end{Bmatrix} \\ &\quad \times \int d_0(r) \frac{f_{0l}^l(r, R)}{rR} dr \delta(\mathbf{r}_b - \frac{m_A}{m_B}\mathbf{r}_a)\end{aligned}$$

The radial part of the form factor is given

$$\begin{aligned}F_{lsj}(r_a) &= \int d_0(r) \frac{f_{0l}^l(r, R)}{rR} d\mathbf{r} \\ &= N_3 \sqrt{4\pi} (\frac{3}{2}\kappa^2)^{-3/2} \frac{1 + (-)^{l_1 + l_2 - l}}{2} \sqrt{\frac{(2l_1+1)(2l_2+1)}{2(2l+1)(1 + \delta_{n_1 n_2}\delta_{l_1 l_2}\delta_{j_1 j_2})}} \\ &\times \int_0^\infty \exp(-X^2) X^2 \int_{-1}^{+1} u_{l_1 j_1}(r_1) u_{l_2 j_2}(r_2) [(l_1 l_2 00 | l0) d_{0,0}^{l_1}(\theta_1) d_{0,0}^{l_2}(\theta_2) \\ &\qquad + 2 \sum_{m>0} (l_1 l_2 m, -m | l0) d_{m,0}^{l_1}(\theta_1) d_{-m,0}^{l_2}(\theta_2)] dx \; dX\end{aligned}$$

The D_0 value is given as follows in the cluster model with Gaussian interactions. The cluster wave function and interaction are defined

$$\phi(\boldsymbol{\rho}) = (\frac{4\kappa^2}{\pi})^{3/4} \exp(-2\kappa^2 \rho^2), \qquad V(\boldsymbol{\rho}) = 2V_0 \exp(-\beta^2 \rho^2).$$

Then the D_0^C value is

$$D_0^C = \int \phi(\boldsymbol{\rho}) V(\boldsymbol{\rho}) d\boldsymbol{\rho} = 2V_0 (\frac{\pi}{2\kappa^2+\beta^2})^{3/2} (\frac{4\kappa^2}{\pi})^{3/4}.$$

The $d_0(r)$ is given using this D_0^C value

$$\begin{aligned} d_0(r) &= N_3 \exp(-\frac{3}{2}\kappa^2 r^2) \int \exp(-2\kappa^2\rho^2) 2V_0 \exp(-\beta^2\rho^2) d\boldsymbol{\rho} \\ &= 2V_0 (\frac{\pi}{2\kappa^2+\beta^2})^{3/2} N_3 \exp(-\frac{3}{2}\kappa^2 r^2) = D_0^C (\frac{\pi}{4\kappa^2})^{3/4} N_3 \exp(-\frac{3}{2}\kappa^2 r^2) \end{aligned}$$

For the zero-range constant the above $D_0^{C2}(\frac{\pi}{4\kappa^2})^{3/2}$ value must be input.

If more refined treatment about the interaction is used [6]

$$\begin{aligned} V_{ib}(\mathbf{r}_{ib}) &= V_0 \exp(-\beta^2 r_{ib}^2) \\ d_0(r) &= V_0 N_3 \int \exp[-\kappa^2(\frac{3}{2}r^2 + 2\rho^2)] \exp[-\beta^2(\frac{1}{4}r^2+\rho^2)] \\ &\quad \times [\exp(-\beta^2 \mathbf{r}\cdot\boldsymbol{\rho}) + \exp(\beta^2 \mathbf{r}\cdot\boldsymbol{\rho})] d\boldsymbol{\rho} \\ &= 2V_0 (\frac{\pi}{2\kappa^2+\beta^2})^{3/2} N_3 \exp[-\frac{3}{2}\kappa^2 (\frac{6\kappa^2+4\beta^2}{6\kappa^2+3\beta^2}) r^2] \end{aligned}$$

Following values must be input for the zero-range constant, the range parameter and the normalization constant of triton(or ^{3}He) particle

$$D_0^2 = D_0^{C2} (\frac{\pi}{4\kappa^2})^{3/2}, \qquad K' = \sqrt{\frac{6\kappa^2+4\beta^2}{6\kappa^2+3\beta^2}}\ \kappa, \qquad N_3' = (\frac{6\kappa^2+3\beta^2}{6\kappa^2+4\beta})^{3/2} N_3$$

In the case of (α,d) reaction the projectile wave functions must be changed as follows

$$\begin{aligned} \phi_\alpha &= N_\alpha \exp[-\kappa^2 \sum_{ij} (\mathbf{r}_i - \mathbf{r}_j)^2] = N_\alpha \exp[-\kappa^2(2r^2 + 2r'^2 + 4\rho^2)] \\ \phi_d &= N_d \exp(-\eta^2 r'^2), \\ N_\alpha &= 2^{3/4} (\frac{4\kappa^2}{\pi})^{9/4}, \qquad N_d = (\frac{2\eta^2}{\pi})^{3/4}. \end{aligned}$$

We introduce the zero-range approximation

$$\int \phi_\alpha [V_{13}(r_{13}) + V_{14}(r_{14}) + V_{23}(r_{23}) + V_{24}(r_{24})] \phi_d d\mathbf{r}' \approx d_0(\mathbf{r}) \delta(\boldsymbol{\rho}) = J^{-1} d_0(\mathbf{r}) \delta(\mathbf{r}_b - \frac{m_A}{m_B}\mathbf{r}_a)$$

In this program the $d_0(r)$ has the most simple form

$$d_0(\mathbf{r}) = N_\alpha N_d (\frac{\pi}{2\kappa^2+\eta^2})^{3/2} \exp(-2\kappa^2 r^2)$$

The D_0 value of the cluster model with Gaussian interaction is given

$$D_0^C = \int (\frac{8\kappa^2}{\pi})^{3/4} \exp(-4\kappa^2\rho^2) 4V_0 \exp(-\beta^2\rho^2) d\boldsymbol{\rho} = 4V_0 (\frac{\pi}{4\kappa^2+\beta^2})^{3/2} (\frac{8\kappa^2}{\pi})^{3/4}$$

The $d_0(r)$ is given using the D_0^C value

$$\begin{aligned} d_0(r) &= \int N_\alpha \exp[-\kappa^2(2r^2+2r'^2+4\rho^2)]4V_0\exp(-\beta^2\rho^2)N_d\exp(-\eta^2 r'^2)d\mathbf{r}'\ d\boldsymbol{\rho} \\ &= D_0^C(\frac{\pi}{8\kappa^2})^{3/4}N_\alpha N_d(\frac{\pi}{2\kappa^2+\beta^2})^{3/2}\exp(-2\kappa^2 r^2) \end{aligned}$$

For the zero-range constant the above $D_0^{C2}(\frac{\pi}{8\kappa^2})^{3/2}$ value must be input.

2.5.2.6 Spectroscopic amplitude of projectile system

We assume all nucleons of the projectile system are in 0s-state.
One-nucleon transfer reaction

reaction	$(\tau_b\frac{1}{2}\nu_b\nu \mid \tau_a\nu_a)$	$\sqrt{n_a}$	$\langle n_b(s_b\tau_b); s\frac{1}{2} \mid\}n_a s_a\tau_a\rangle$	spectrosco. amp.
(d,p)	$-1/\sqrt{2}$	$\sqrt{2}$	1	$=-1$
(d,n)	$1/\sqrt{2}$	$\sqrt{2}$	1	$=+1$
(h,d)	1	$\sqrt{3}$	$-1/\sqrt{2}$	$=-\sqrt{3/2}$
(t,d)	1	$\sqrt{3}$	$1/\sqrt{2}$	$=-\sqrt{3/2}$
(α,h)	$-1/\sqrt{2}$	$\sqrt{4}$	1	$=-\sqrt{2}$
(α,t)	$1/\sqrt{2}$	$\sqrt{4}$	1	$=+\sqrt{2}$

Two-nucleon transfer reaction

reaction	$(\tau_b\tau\nu_b\nu \mid \tau_a\nu_a)$	$\sqrt{\binom{n_a}{2}}$	$\langle n_b(s_b\tau_b); st \mid\}n_a s_a\tau_a\rangle$	spectrosco. amp.
(t,p)	$-1/\sqrt{2/3}$	$\sqrt{3}$	$-1/\sqrt{2}$	$=+1$
(h,p)(s=0,t=1)	$-1/\sqrt{3}$	$\sqrt{3}$	$-1/\sqrt{2}$	$=1/\sqrt{2}$
(h,p)(s=1,t=0)	1	$\sqrt{3}$	$1/\sqrt{2}$	$=\sqrt{3/2}$
(h,n)	$\sqrt{2/3}$	$\sqrt{3}$	$-1/\sqrt{2}$	$=-1$
(t,n)(s=0,t=1)	$1/\sqrt{3}$	$\sqrt{3}$	$-1/\sqrt{2}$	$=-1/\sqrt{2}$
(t,n)(s=1,t=0)	1	$\sqrt{3}$	$1/\sqrt{2}$	$=\sqrt{3/2}$
(α,d)	1	$\sqrt{6}$	$-1/\sqrt{2}$	$=-\sqrt{3}$

2.5.2.7 Integration step size of the form factor

In this program the step size of integration for each channel is fixed on the basis of initial channel. In the case of reading the external form factors, one must be careful

about the choise of the step size of the form factor. The step size of the form factor and normalization factor of integration of overlap integral for each step are shown below,

one-step process

process	step size	factor
stripping	$\triangle r_i$	$\frac{m_f}{m_i}$
pick-up	$\frac{m_i}{m_f}\triangle r_i$	$(\frac{m_i}{m_f})^2$

two-step process

first-step	step size	factor
stripping	$\triangle r_i$	$\frac{m_m}{m_i}$
pick-up	$\frac{m_i}{m_m}\triangle r_i$	$(\frac{m_i}{m_m})^2$

second-step	step size	factor
stripping	$\frac{m_i}{m_m}\triangle r_i$	$\frac{m_f}{m_m}\frac{m_i}{m_m}$
pick-up	$\frac{m_i}{m_f}\triangle r_i$	$\frac{m_m}{m_f}\frac{m_i}{m_f}$

2.6 Local energy Approximation

2.6.1 Non-locality correction

The non-locality of the distorted wave and/or bound state wave function can be corrected for approximately by introducing form factor corrections. The non-locality has the effect on the wave function in the nuclear interior; the wave function for non-local potential is reduced inside the nucleus compared to that for a local potential which gives the same scattering. This reduction can be well represented by a damping factor obtained from the local energy approximation [8]. This factor is of the form

$$H(r) = \exp[\frac{\beta_i^2}{8}\frac{2\mu_i}{\hbar^2}V_i(r)]$$

where β_i's are the non-locality parameters, the μ_i's are the reduced masses and the V_i's are the equivalent local potentials for each of projectiles and bound state particle. In the case of a bound state the factor H is multiplied on the bound state wave function and then the function is renormalized to unity. In this program the V_i do not include the spin-orbit part of the optical potential for the projectile but include all of central, spin-orbit and Coulomb potential for the bound state particles. Typical values of β parameter are

particle	$\beta(fm)$
p	0.85
d	0.54
^{3}He	$0.2 \sim 0.5$
α	0.2

2.6.2 Finite-range correction

The zero-range approximation tends to overestimate the contributions from the nuclear interior. The local energy approximation yields a simple radial correction factor for the form factor of zero-range approximation [8].

For the stripping reaction, a(b+x) + A → b + B(A+x), one can write the transition amplitude as

$$T_{fi} = \int d\,\mathbf{R} \int d\,\mathbf{r} D(\mathbf{r}) \chi_f^{(+)*}(\mathbf{R}+\mathbf{r}) \phi_x^*(\mathbf{R}) \chi_i^{(-)}(\mathbf{R} + \frac{m_b}{m_a}\mathbf{r})$$

where $D(\mathbf{r})$ is the overlap function between a and b,

$$D(\mathbf{r}) = D(\mathbf{r}_{bx}) = V_{bx}(\mathbf{r}_{bx}) \phi_{bx}(\mathbf{r}_{bx}).$$

If we define

$$G(K) = \int \exp(i\mathbf{K}\cdot\mathbf{r}) D(\mathbf{r}) d\,\mathbf{r}$$

the zero-range normalization is given by $D_0 = G(0)$ and the finite range correction parameter R is given

$$R^2 = -\frac{1}{G(K^2)} \frac{\partial G(K^2)}{\partial (K^2)}_{K^2=0}$$

The first order correction factor with the aid of the local energy approximation for $D(\mathbf{r})$ is

$$H(r) = [1+\Lambda(r)]^{-1} \qquad \text{Hulthén form} \quad D^H(r) = D_0^H \frac{1}{4\pi R^2} \frac{\exp(-r/R)}{r},$$

$$H(r) = \exp[-\Lambda(r)] \qquad \text{Gaussian form} \quad D^G(r) = D_0^G (\frac{1}{4\pi R^2})^{3/2} \exp[-(r/R)^2]$$

and

$$\Lambda(r) = \frac{2}{\hbar^2} \frac{m_b m_x}{m_a} R^2 \left[E_b - V_b(r_b) + E_x - V_x(r_x) - E_a + V_a(r_a)\right]$$

where E, V and m are the energy, potential and mass of each light particle.

A positive FNRNG will select the Hulthén form of $H(r)$ and a negative FNRNG will select the Gaussdian form of $H(r)$. Typical values of R parameter are

reaction	$R(fm)$
(p,d)	$0.621 \sim 0.695$
(^{3}He,d)	0.770
(t,d)	0.845
(α,^{3}He)	0.7

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1 Difference from the program TWOSTP

This program is the finite-range form factor version of the program **TWOSTP**. Here the differences from the code **TWOSTP** for the input data and the matters to be attended are pointed out.

(1) The number of the intermediate channel is limited up to one.

(2) If the finite-range form factors are used, one can not calculate both one-step and two-step processes simultaneouly. One must calculate separately and sum up the amplitudes by the aid of the subroutine MIX.

(3) The modification of card 1.0 basic data card.

`1.0`	
`NUBCHN`	number of intermediate channels. (NUBCHN ≤ 1)
`RMAX`	
`NRMIN`	
`NRMAX`	number of mesh points. $\triangle r = \frac{\text{RMAX}}{\text{NRMAX}}$, (NRMAX ≤ 100)
`ELABI`	
`KTZF(1)`	control integer of the first-step form factor. =0; zero-range form factor, =1; finite-range form factor, =2; same as former case, =3; reads in from permanent file, =4; writes on the permanent file.
`KTZF(2)`	same control integer of the second step.
`1.1`	
`MESH`	distorted waves are calculated with mesh $\triangle r$/MESH, but overlap integrals are calculated with $\triangle r$. (NRMAX×MESH ≤ 300)

In the case to calculate the nonorthogonality term the optical potential strength VD(□) and WD(□) (in card 5.□) of the intermediate channel must be negative. In this case δ-function is used instead of Green function.

2 Input Data for finite-range form factor

2.1 Control Data

FORMAT(10I2)

KTRL(1)	Type of the wave function for the system 1. =0; depth search, =1; binding energy search, from the next, the wave function with the analytic form =2; Gaussian $N_G \exp[-(r/\alpha)^2]$ =3; Yukawa $N_Y \exp(-\alpha/r)$ =4; Hulthen $N_H[\exp(-\alpha r) - \exp(-\beta r)]/r$ =5; two-Gaussian $\alpha_1 \exp[-(r/\alpha)^2] + \beta_2 \exp[-(r/\beta)^2]$ =6; constant =WREAD(1) =7; local-Gaussian $N_{GL} \exp[-((r-\alpha)/\beta)^2]$
KTRL(2)	type of the binding potential for the system 1. If KTRL(2) ≥ 4 the wave function is multiplied by the searched binding potential. (U_n; nuclear part, U_c; Coulomb part) 0 1 2 3 $\phi(r_1)$ 4 6 8 10 $\phi(r_1) \times U_n(r_1)$ 5 7 9 11 $\phi(r_1) \times (U_n + U_c)$ Woods-Saxon Gaussian Yukawa Hulthén Woods-Saxon potential $V_1 \times [1 + \exp((r-R)/a)]^{-1}$ Gaussian potential $V_1 \times \exp[-(r/\alpha)^2]$ Yukawa potential $V_1 \times \exp(-\alpha_1 r)/r$ Hulthen potential $V_1 \times \exp(-\beta_1 r)/[\exp(-\alpha_1 r) - \exp(-\beta_1 r)]$
KTRL(3)	same control integer for the system 2 as KTRL(1). If KTRL(3)=6 the wave function has the constant value (=WREAD(8)).
KTRL(4)	same control integer for the system 2 as KTRL(2).
KTRL(5)	type of the reaction interaction =0; constant=1.0, =1; not used =2; Gaussian, $V_0 \times \exp[-(r/\xi)^2]$ =3; Yukawa, $V_0 \times \exp(-\xi r)/r$ =4; Woods-Saxon, $V_0 \times [1 + \exp((r-R_D)/\xi)]^{-1}$ =5; Local-Gaussian, $V_0 \times \exp[-((r-R_D)/\xi)^2]$
KTRL(6)	type of the reaction =0; stripping or pick-up reaction =1; exchange or knock-out reaction =3; other type of reaction
KTRL(7)	is not used.
KTRL(8)	calculates the nonorthogonality form factor with the operator method, =0; no, =1; yes.
KTRL(9)	prints out 2-dimensional radial form factors every n-th point =0; no, =n; yes.
KTRL(10)	prints out bound state wave functions for the system 1 and 2, =0; no, =1; yes.

2.2 Numerical Data

FORMAT(8F10.5)

1.0	
AMP	a_{lsj}; spectroscopic amplitude.
NRANGE	number of mesh points of r_b (NRANGE ≤ 30) $\mid \frac{s_3}{t_3} \mid r_a - \text{NRANGE}\triangle r_b \leq r_b \leq \mid \frac{s_3}{t_3} \mid r_a + \text{NRANGE}\triangle r_b$
NAFAC	factor by which the original mesh size of r_a is reduced after interpolation is made.
NBFAC	same factor for r_b.
KAPG	maximum number of k for $g_k(r_a, r_b)$ =0, program calculates this number. $=k_{max}$, puts this number.
2.0	
	quantum numbers of the bound particles.
IREAD(2)	n_1 ; number of nodes
IREAD(3)	l_1 ; orbital angular momentum. for the system 1.
IREAD(4)	j_1 ; total angular momentum.
IREAD(5)	n_2 ; number of nodes
IREAD(6)	l_2 ; orbital angular momentum. for the system 2.
IREAD(7)	j_2 ; total angular momentum.
IREAD(8)	s_x ; spin of the transferred particle.
3.0	
FREAD(1)	V_0 ; strength of the reaction interaction.
FREAD(2)	ξ ; range parameter of the reaction interaction.
FREAD(3)	$\left.\begin{array}{l}\varepsilon_1 \text{ ; binding energy or} \\ V_1 \text{ ; depth of binding potential}\end{array}\right\}$ for the system 1.
FREAD(4)	$\left.\begin{array}{l}\varepsilon_2 \text{ ; binding energy or} \\ V_2 \text{ ; depth of binding potential}\end{array}\right\}$ for the system 2.
4.0	
	Factor of coordinate transformation, $\mathbf{r}_i = s_i\mathbf{r}_a + t_i\mathbf{r}_b$.
FREAD(15)	s_1 If KTRL(6)=0 or 1 and
FREAD(16)	t_1 FREAD(16)=FREAD(17)=FREAD(18)=0.0
FREAD(17)	s_2 s's and t's are calculated in the program.
FREAD(18)	t_2 If KTRL(6)=0 and FREAD(15)=1.0, $\mathbf{r}_3 = \mathbf{r}_1$ or
FREAD(19)	s_3 If KTRL(6)=0 and FREAD(15)=2.0, $\mathbf{r}_3 = \mathbf{r}_2$.
FREAD(20)	t_3 If KTRL(6)=1 and FREAD(15)=1.0 (2.0) exact(approximate i.e., $\mathbf{r}_a = \mathbf{r}_1$ and $\mathbf{r}_b = \mathbf{r}_2$) calculation is made

5.0	
	Binding potential parameters for the system 1.
WREAD(1)	m_1 or α
WREAD(2)	M_{C1} or β for analytic form.
WREAD(3)	$z_x Z_{C1}$
WREAD(4)	r_{01} or α_1 binding potential parameter.
WREAD(5)	a_1 or β_1 for non Woods-Saxon type.
WREAD(6)	V_{so1}
6.0	
	Binding potential parameters for the system 2.
WREAD(8)	m_2 or α
WREAD(9)	M_{C2} or β for analytic form.
WREAD(10)	$z_x Z_{C2}$
WREAD(11)	r_{02} or α_1 binding potential parameter.
WREAD(12)	a_2 or β_1 for non Woods-Saxon type.
WREAD(13)	V_{so2}
7.0	
WREAD(15)	R_D ; radius of the Woods-Saxon or local-Gaussian reaction interaction.
WREAD(16)	s_4
WREAD(17)	t_4

The case to read the bound state wave function of the system □.

8.□ DRX(□) KT	Δr ; step size of the bound state wave function of system □. if KT> 0, uses the same wave function just as the run before.
(5E15.8)	example FORMAT card.
FFR(I,□) I=1,400	Bound state wave function for the system □. if DRX(□) = 0.0 $\triangle r = \dfrac{1.5 \times \mathrm{RMAX}}{400}$

A blank card will terminate to read form factor data.

blank card

3 Description of TWOFNR

This program is the finite-range form factor version of the program **TWOSTP**.

3.1 Finite-range Form Factor

The matrix elements of interaction causing the reaction, taken between the internal states of colliding pair in the step of reaction processes, can be expanded into terms which correspond to the transfer to the nucleus of a definite angular momenta $\boldsymbol{l}$, $\boldsymbol{s}$ and $\boldsymbol{j}(\boldsymbol{l}+\boldsymbol{s}=\boldsymbol{j})$,

$$\begin{aligned}&\langle I_B M_B, s_b\sigma_b|V|I_A M_A, s_a\sigma_a\rangle\\ &\quad=\sum_{lsj} i^{-l}(-)^{s_b-\sigma_b}(I_A j M_A, M_B-M_A|I_B M_B)(s_a s_b\sigma_a,-\sigma_b|s\sigma_a-\sigma_b)\\ &\qquad\times(lsm,\sigma_a-\sigma_b|jM_A-M_B)A_{lsj}f_{lsj,m}(\mathbf{r}_a,\mathbf{r}_b),\end{aligned}$$

where $m=M_B-M_A+\sigma_b-\sigma_a$.

If we assume that the reaction is due to central interaction, form factor $f_{lsj,m}(\mathbf{r}_a,\mathbf{r}_b)$ has the common form, for all particle transfer, knock-on and inelastic scattering processes.

$$\begin{aligned}f_{lsj,m}(\mathbf{r}_a,\mathbf{r}_b) &= J\sum_{l_1 l_2}\ (\text{kinematical factors})\ f^{l_1 l_2}_{l,m}(\mathbf{r}_1,\mathbf{r}_2),\\ f^{l_1 l_2}_{l,m}(\mathbf{r}_1,\mathbf{r}_2) &= \sum_{m_1 m_2}(l_1 l_2 m_1,-m_2|lm)Y^{m_1*}_{l_1}(\hat{r}_1)Y^{-m_2*}_{l_2}(\hat{r}_2)\\ &\qquad\times u_{l_1}(r_1)V(r_3)u_{l_2}(r_2),\end{aligned}$$

where $u_{l_1}(r_1)$ and $u_{l_2}(r_2)$ are the nuclear bound state wave functions, which have the angular momentum l_1 and l_2, respectively, and $V(r_3)$ is the nuclear interaction. Here the vector $\mathbf{r}_i$ of system i is represented by the linear combination of the channel vectors $\mathbf{r}_a$ and $\mathbf{r}_b$,

$$\mathbf{r}_i=s_i\mathbf{r}_a+t_i\mathbf{r}_b.$$

J is the Jacobian for the transformation from the natural vector coordinate system ($\mathbf{r}_1$, $\mathbf{r}_2$) to the channel vector coordinate system ($\mathbf{r}_a$, $\mathbf{r}_b$),

$$J=[s_1t_2-s_2t_1]^3.$$

This form factor transforms like Y^{m*}_l and can be expanded with the angular momenta of the distorted waves of prior and post channels,

$$\begin{aligned}f^{l_1 l_2}_{l,m}(\mathbf{r}_1,\mathbf{r}_2) &= \sum_{l_a l_b}F^{l_1 l_2}_{l,l_a l_b}(r_a,r_b)\\ &\qquad\times\sum_{m_a m_b}(l_a l_b m_a,-m_b|lm)Y^{m_a*}_{l_a}(\hat{r}_a)Y^{-m_b*}_{l_b}(\hat{r}_b).\end{aligned}$$

By inversion we have

$$F^{l_1 l_2}_{l,l_a l_b}(r_a, r_b) = \int\int f^{l_1 l_2}_{l,m}(\mathbf{r}_1, \mathbf{r}_2) \sum_{m_a m_b} (l_a l_b m_a, -m_b | l m) Y^{m_a}_{l_a}(\hat{r}_a) Y^{-m_b}_{l_b}(\hat{r}_b) d\mathbf{r}_a d\mathbf{r}_b.$$

The scalar radial part of the form factor can be expanded

$$u_{l_1}(r_1) V(r_3) u_{l_2}(r_2) = \sum_k \frac{2k+1}{2} g_k(r_a, r_b) P_k(\mu)$$

where $P_k(\mu)$ is the Legendre polynomial with the argument $\mu = (\hat{r}_a \cdot \hat{r}_b)$, which is the cosine between the vectors $\mathbf{r}_a$ and $\mathbf{r}_b$. Here, by inversion,

$$g_k(r_a, r_b) = \int_{-1}^{1} u_{l_1}(r_1) V(r_3) u_{l_2}(r_2) P_k(\mu) d\mu.$$

The basic transformation used here is the one which converts the spherical harmonic $Y^m_l(\hat{r})$, where $\mathbf{r} = s\mathbf{r}_a + t\mathbf{r}_b$, into spherical harmonics in $\mathbf{r}_a$ and $\mathbf{r}_b$ separately,

$$\begin{aligned} r^l Y^m_l(\hat{r}) &= \sum_{\lambda\mu} \sqrt{\frac{4\pi}{2\lambda+1}\binom{2l+1}{2\lambda}} (s r_a)^{l-\lambda} (t r_b)^{\lambda} \\ &\quad \times (l-\lambda \lambda m-\mu, \mu | l m) Y^{m-\mu}_{l-\lambda}(\hat{r}_a) Y^{\mu}_{\lambda}(\hat{r}_b), \end{aligned}$$

where λ runs from 0 to l, and

$$\binom{a}{b} = \frac{a!}{b!(a-b)!}$$

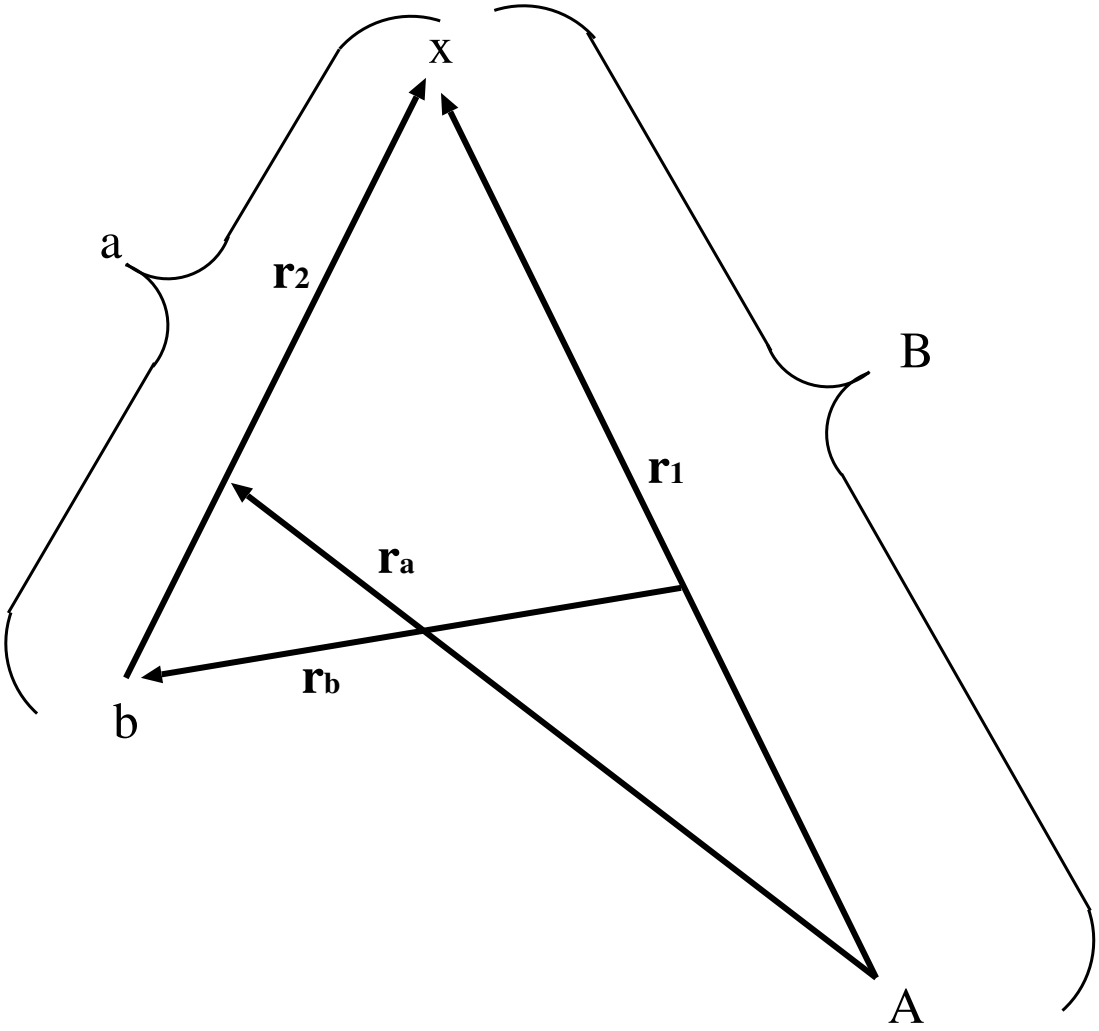
is the binomial coefficient.

The final expression is given

$$\begin{aligned} F^{l_1 l_2}_{l,l_a l_b}(r_a, r_b) &= \frac{1}{2} \sum_{\lambda\lambda'\Lambda_a\Lambda_b k} \sqrt{\binom{2l_1+1}{2\lambda}\binom{2l_2+1}{2\lambda'}} \\ &\times (s_1 r_a)^{l_1-\lambda} (s_2 r_a)^{l_2-\lambda'} (t_1 r_b)^{\lambda} (t_2 r_b)^{\lambda'} \\ &\times (2k+1) \sqrt{(2l_a+1)(2l_b+1)(2l_1+1)(2l_2+1)(2l_1-2\lambda+1)(2l_2-2\lambda'+1)} \\ &\times (l_a k 00 | \Lambda_a 0)(l_1 - \lambda l_2 - \lambda' 00 | \Lambda_a 0)(l_b k 00 | \Lambda_b 0)(\lambda\lambda' 00 | \Lambda_b 0) \\ &\times \begin{Bmatrix} l_1-\lambda & l_2-\lambda' & \Lambda_a \\ \lambda & \lambda' & \Lambda_b \\ l_1 & l_2 & l \end{Bmatrix} \\ &\times (-)^{l_a+l_b+l+k} W(l_a \Lambda_a l_b \Lambda_b; k l) g_k(r_a, r_b). \end{aligned}$$

3.2 Transfer Reaction

3.2.1 Stripping Reaction



The nuclear matrix element of the internal wave functions of the colliding pair is given

$$\langle I_B M_B, s_b m_b | V | I_A M_A, s_a m_a \rangle = \int \langle I_B M_B | I_A M_A \rangle \langle s_b m_b | V(r_2) | s_a m_a \rangle d\boldsymbol{\xi}_x$$

where $\boldsymbol{\xi}_x$ denotes the internal coordinate of the transferred particle x and the post representation is used. The overlap functions of the target and projectile system are

$$\begin{aligned}\langle I_B M_B | I_A M_A \rangle \;=\; & \sum_{l_1 s_x j, m_1 m_x} S^{(1)}_{I_B I_A l_1 s_x j}(I_A j M_A, M_B - M_A | I_B M_B) \\ & \times (l_1 s_x m_1 m_x | j M_B - M_A) u_{l_1 j}(r_1) i^{-l_1} Y_{l_1}^{m_1 *}(\hat{r}_1) \psi_{s_x}^{m_x *}(\boldsymbol{\xi}_x)\end{aligned}$$

$$\langle s_b m_b|V(r_2)|s_a m_a\rangle = \sum_{l_2 s_x s, m_2 m_x} S^{(2)}_{s_a s_b l_2 s_x s}(s_b s m_b, m_a - m_b|s_a m_a) \times(l_2 s_x m_2 m_x|s m_a - m_b)V(r_2)u_{l_2 s}(r_2)i^{l_2}Y^{m_2}_{l_2}(\hat{r}_2)\psi^{m_x}_{s_x}(\boldsymbol{\xi}_x).$$

Then the final result is given

$$\begin{aligned}&\langle I_B M_B, s_b m_b|V|I_A M_A, s_a m_a\rangle \\ &= \sum_{lsj}\sum_{l_1 l_2 s_x} S^{(1)}_{I_B I_A l_1 s_x j} S^{(2)}_{s_a s_b l_2 s_x s} \\ &\times i^{l_2-l_1+l}(-)^{s_x-s}\sqrt{(2l+1)(2s+1)}\ W(l_1 l_2 j s; l s_x) \\ &\times(-)^{s_b-m_b}(I_A j M_A, M_B - M_A|I_B M_B)(s_a s_b m_a, -m_b|s m_a - m_b) \\ &\times(lsm, m_a - m_b|j M_B - M_A)\sqrt{\frac{2s_a+1}{2s+1}}\ i^{-l}\ f^{l_1 j l_2 s}_{lm}(r_1, r_2),\end{aligned}$$

where

$$f^{l_1 j l_2 s}_{lm}(r_1, r_2) = \textstyle\sum_{m_1 m_2}(l_1 l_2 m_1, -m_2|l m_1 - m_2)Y^{m_1 *}_{l_1}(\hat{r}_1)Y^{-m_2 *}_{l_2}(\hat{r}_2) \times u_{l_1 j}(r_1)V(r_2)u_{l_2 s}(r_2).$$

The spectroscopic amplitude, which must be input, is

$$a_{lsj} = S^{(1)}_{I_B I_A l_1 s_x j} S^{(2)}_{s_a s_b l_2 s_x s}.$$

$$\text{(kinematical factor)} = i^{l_2-l_1+l}(-)^{s_x-s}\sqrt{(2l+1)(2s+1)}\ W(l_1 l_2 j s; l s_x),$$

this kinematical factor is taken into account in subroutine FFGENF.

The factors s_i and t_i are given

$$s_1 = \frac{m_a m_B}{m_x m_T}, \qquad t_1 = -\frac{m_b m_B}{m_x m_T},$$

$$s_2 = \frac{m_a m_A}{m_x m_T}, \qquad t_2 = -\frac{m_a m_B}{m_x m_T},$$

where $m_x m_T = (m_a - m_b)(m_a + m_A)$.

The interaction vector $\mathbf{r}_3$ depends upon the representation of the nuclear matrix element,

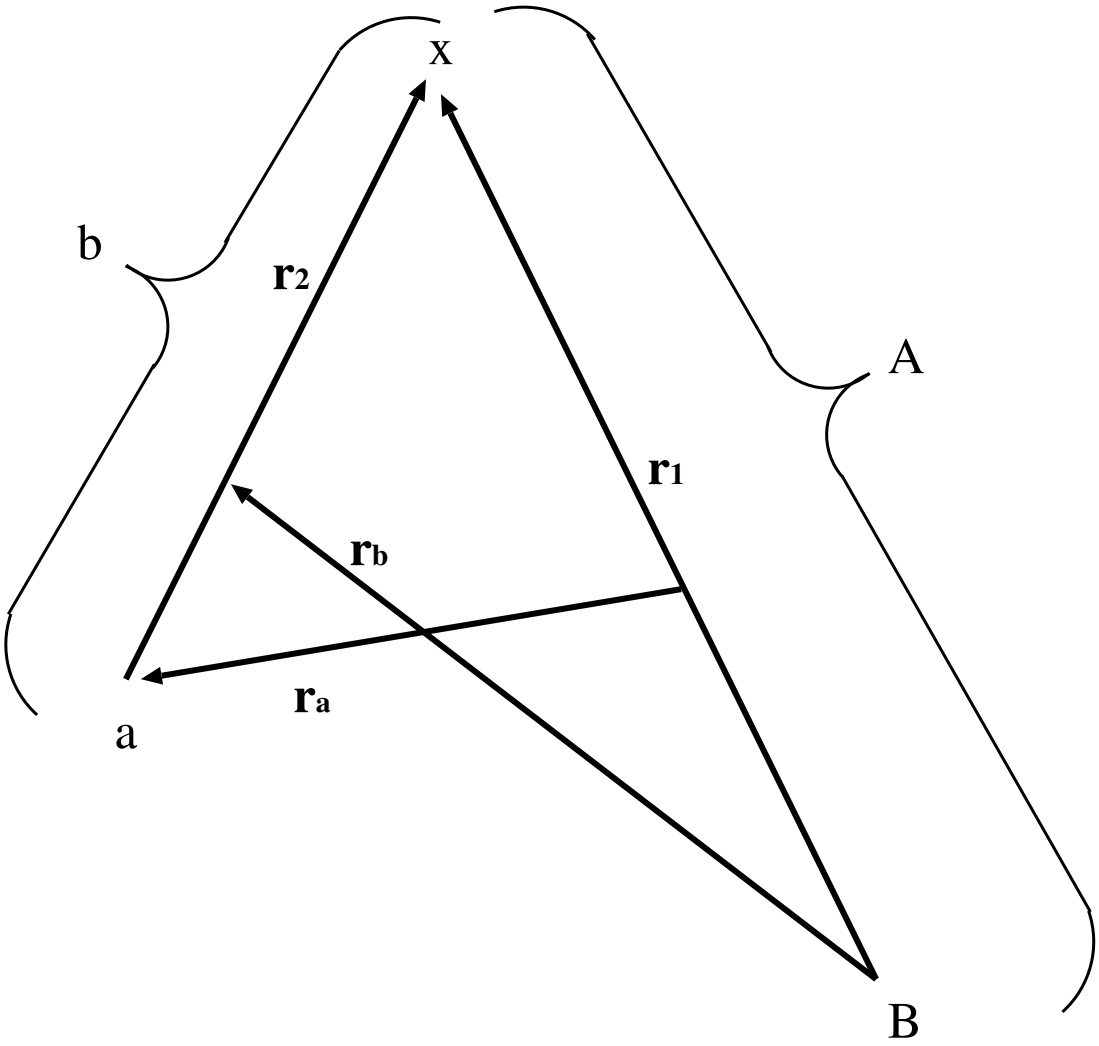
$\mathbf{r}_3 = \mathbf{r}_2$ $\quad (s_3 = s_2, t_3 = t_2)$ $\quad$ for the post representation,

$\mathbf{r}_3 = \mathbf{r}_1$ $\quad (s_3 = s_1, t_3 = t_1)$ $\quad$ for the prior representation.

Jacobian is

$$J = \left[\frac{m_a m_B}{m_x m_T}\right]^3$$

3.2.2 Pick-up Reaction



The nuclear matrix element of the internal wave functions of the colliding pair is given

$$\langle I_B M_B, s_b m_b | V | I_A M_A, s_a m_a \rangle = \int \langle I_B M_B | I_A M_A \rangle \langle s_b m_b | V(r_2) | s_a m_a \rangle d\boldsymbol{\xi}_x$$

where $\boldsymbol{\xi}_x$ denotes the internal coordinate of the transferred particle x and the post representation is used. The overlap functions of the target and projectile system are

$$\begin{aligned}\langle I_B M_B | I_A M_A \rangle &= \sum_{l_1 s_x j, m_1 m_x} S^{(1)}_{I_A I_B l_1 s_x j}(I_B j M_B, M_A - M_B | I_A M_A) \\ &\quad \times (l_1 s_x m_1 m_x | j M_A - M_B) u_{l_1 j}(r_1) i^{l_1} Y^{m_1}_{l_1}(\hat{r}_1) \psi^{m_x}_{s_x}(\boldsymbol{\xi}_x)\end{aligned}$$

$$\begin{aligned}\langle s_b m_b | V(r_2) | s_a m_a \rangle &= \sum_{l_2 s_x s, m_2 m_x} S^{(2)}_{s_b s_a l_2 s_x s}(s_a s m_a, m_b - m_a | s_b m_b) \\ &\quad \times (l_2 s_x m_2 m_x | s m_b - m_a) V(r_2) u_{l_2 s}(r_2) i^{-l_2} Y^{m_2 *}_{l_2}(\hat{r}_2) \psi^{m_x *}_{s_x}(\boldsymbol{\xi}_x).\end{aligned}$$

Then the final result is given

$$
\begin{aligned}
&\langle I_B M_B, s_b m_b | V | I_A M_A, s_a m_a \rangle \\
&= \sum_{lsj} \sum_{l_1 l_2 s_x} S^{(1)}_{I_A I_B l_1 s_x j} S^{(2)}_{s_b s_a l_2 s_x s} \\
&\quad \times i^{l_2 - l_1 + l} (-)^{s_x - s} \sqrt{(2l+1)(2s+1)}\ W(l_1 l_2 j s; l s_x) \\
&\quad \times (-)^{I_A + j - I_B + s_b + s - s_a} \sqrt{\frac{(2I_A+1)(2s_b+1)}{(2I_B+1)(2s_a+1)}} \\
&\quad \times (-)^{s_b - m_b} (I_A j M_A, M_B - M_A | I_B M_B)(s_a s_b m_a, -m_b | s m_a - m_b) \\
&\quad \times (lsm, m_a - m_b | j M_B - M_A) \sqrt{\frac{2s_a+1}{2s+1}}\ i^{-l}\ f^{l_1 j l_2 s}_{lm}(r_1, r_2),
\end{aligned}
$$

where

$$
\begin{aligned}
f^{l_1 j l_2 s}_{lm}(r_1, r_2) \quad = \quad & \sum_{m_1 m_2} (l_1 l_2 m_1, -m_2 | l m_1 - m_2) Y^{m_1 *}_{l_1}(\hat{r}_1) Y^{-m_2 *}_{l_2}(\hat{r}_2) \\
& \times u_{l_1 j}(r_1) V(r_2) u_{l_2 s}(r_2).
\end{aligned}
$$

The spectroscopic amplitude, which must be input, is

$$
a_{lsj} = S^{(1)}_{I_A I_B l_1 s_x j} S^{(2)}_{s_b s_a l_2 s_x s}.
$$

$$
\text{(kinematical factor)} = i^{l_2 - l_1 + l} (-)^{s_x - s} \sqrt{(2l+1)(2s+1)}\ W(l_1 l_2 j s; l s_x),
$$

$$
\text{(inversion factor)} = (-)^{I_A + j - I_B + s_b + s - s_a} \sqrt{\frac{(2I_A+1)(2s_b+1)}{(2I_B+1)(2s_a+1)}}
$$

these kinematical factor and inversion factor between the pick-up and stripping processes are taken into account in subroutine FFGENF.

The factors s_i and t_i are given

$$
s_1 = -\frac{m_a m_A}{m_x m_T}, \qquad t_1 = \frac{m_b m_A}{m_x m_T},
$$

$$
s_2 = -\frac{m_b m_A}{m_x m_T}, \qquad t_2 = \frac{m_b m_B}{m_x m_T},
$$

where $m_x m_T = (m_b - m_a)(m_a + m_A)$.
The interaction vector $\mathbf{r}_3$ depends upon the representation of the nuclear matrix element,

$$
\mathbf{r}_3 = \mathbf{r}_2 \qquad (s_3 = s_2, t_3 = t_2) \qquad \text{for the post representation,}
$$

$$
\mathbf{r}_3 = \mathbf{r}_1 \qquad (s_3 = s_1, t_3 = t_1) \qquad \text{for the prior representation.}
$$

Jacobian is

$$
J = \left[\frac{m_b m_A}{m_x m_T} \right]^3
$$

3.2.3 Projectile wave function

Projectile wave function may be calculated in the program by specifying the analytical form, KTRL(3)=2,$\cdots$ 7, or solving bound state wave function in some type of the potential. Geometric parameters of the spin-orbit potential is the same as that of Woods-Saxon potential. Bound state wave function may be supplied externally. It has positive rise near the origin. Zeros at the origin and infinity are not counted as number of nodes. One can find more realistic wave functions for (p,d) and (d,t) reactions in the sample data, where D-state mixture is taken into account. Interaction potential can be multiplied to the projectile wave function just by controlling KTRL(4).

3.2.4 Nonorthogonality form factor including the operator

In the two-step process of the particle rearrangement reaction, the nonorthogonality term, so called the "nonorthogonality-nonorthogonality" (hereafter named "non-non") term, appears if the projectile particle of the intermediate channel is lighter than the projectile particles of both initial and final channel. The transition amplitude of "non-non" term is given

$$T^{non-non} = -\langle \chi_f^{(-)} \psi_f | T_m + U_m - E_m | \psi_m \rangle \langle \psi_m | \psi_i \chi_i^{(+)} \rangle$$

where ψ's are the internal wave functions of the colliding pair in each channel and T_m, U_m and E_m are the kinematical operator, the optical potential and the energy of relative motion for the projectile of the intermediate channel, respectively. This type of nonorthogonality form factor, which includes the operator, can be calculated as follows

$$\langle \psi_a | T_m + U_m - E_m | \psi_b \rangle_{l,\mu} = (T_m + U_m - E_m) \langle \psi_a | \psi_b \rangle_{l,\mu}$$

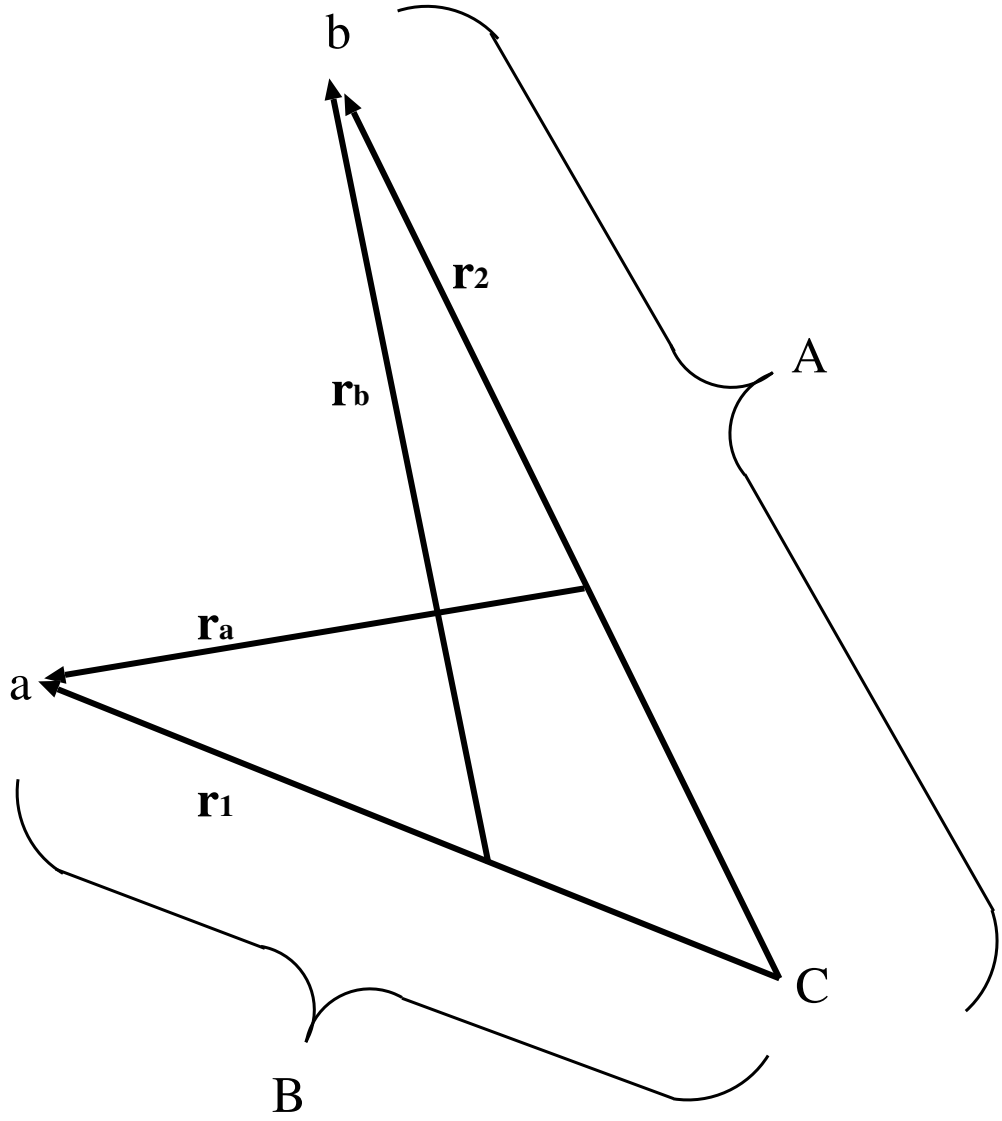
where m is the intermediate channel, which equals a or b, and

$$\langle \psi_a | \psi_b \rangle_{l,\mu} = \sum_{l_a l_b} F^l_{l_a l_b}(r_a, r_b) [Y_{l_a}(\hat{r}_a) \times Y_{l_b}(\hat{r}_b)]^{(l)*}_{\mu}.$$

$$\begin{aligned}
&(T_m + U_m - E_m) \langle \psi_a | \psi_b \rangle_{l,\mu} \\
&= \sum_{l_a l_b} \{ -\frac{\hbar^2}{2\mu} [\frac{1}{r_m^2} \frac{\partial}{\partial r_m} (r_m^2 \frac{\partial}{\partial r_m}) - \frac{l_m(l_m+1)}{r_m^2}] + U_m(r_m) - E_m \} F^l_{l_a l_b}(r_a, r_b) \\
&\quad \times [Y_{l_a}(\hat{r}_a) \times Y_{l_b}(\hat{r}_b)]^{(l)*}_{\mu}
\end{aligned}$$

The second derivative in the above equation is calculated by the numerical differentiation of the usual nonorthogonality form factor. In this program the sign of this form factor is changed opposite because we assume that an attractive potential has the positive sign.

3.3 Knock-out and Exchange Reaction



The initial and final state internal wave function of colliding pairs are given

$$
\begin{aligned}
&|I_A M_A, s_a m_a\rangle = \sum_{I_C l_2 j_2, M_C m_2} S^{(2)}_{I_A I_C s_b l_2 j_2} \\
&\times (I_C j_2 M_C, M_A - M_C | I_A M_A)(l_2 s_b m_2, M_A - M_C - m_2 | j_2 M_A - M_C) \\
&\times u_{l_2 j_2}(r_2) i^{l_2} Y^{m_2}_{l_2}(\hat{r}_2) \psi^{M_A - M_C - m_2}_{s_b}(\xi_b) \psi^{m_a}_{s_a}(\xi_a) \Psi^{M_C}_{I_C}(\xi_{I_C}),
\end{aligned}
$$

$$
\begin{aligned}
&\langle I_B M_B, s_b m_b| = \sum_{I_C l_1 j_1, M_C m_1} S^{(1)}_{I_B I_C s_a l_1 j_1} \\
&\times (I_C j_1 M_C, M_B - M_C | I_B M_B)(l_1 s_a m_1, M_B - M_C - m_1 | j_1 M_B - M_C) \\
&\times u_{l_1 j_1}(r_1) i^{-l_1} Y^{m_1 *}_{l_1}(\hat{r}_1) \psi^{M_B - M_C - m_1 *}_{s_a}(\xi_a) \psi^{m_b *}_{s_b}(\xi_a) \Psi^{M_C *}_{I_C}(\xi_{I_C}).
\end{aligned}
$$

We assume that the interaction causing the reaction process is the central force

$$\begin{aligned} V &= (\boldsymbol{\sigma}_a^{(\sigma)} \cdot \boldsymbol{\sigma}_b^{(\sigma)})(\boldsymbol{\tau}_a^{(\tau)} \cdot \boldsymbol{\tau}_b^{(\tau)}) f(|\mathbf{r}_a - \mathbf{r}_b|) \\ &= (-)^{\sigma+\tau}(2\sigma+1)(2\tau+1)[\boldsymbol{\sigma}_a^{(\sigma)} \times \boldsymbol{\sigma}_b^{(\sigma)}]_0^{(0)}[\boldsymbol{\tau}_a^{(\tau)} \times \tau_b^{(\boldsymbol{\tau})}]_0^{(0)} f(|\mathbf{r}_a - \mathbf{r}_b|) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\sigma}^{(0)} &= 1, & \boldsymbol{\tau}^{(0)} &= 1 \\ \boldsymbol{\sigma}^{(1)} &= \boldsymbol{\sigma}, & \boldsymbol{\tau}^{(1)} &= \boldsymbol{\tau} \end{aligned}$$

The final result is given

$$
\begin{aligned}
&\langle I_B M_B, s_b m_b | V | I_A M_A, s_a m_a \rangle \\
&= \sum_{lsj, I_C l_1 j_1 l_2 j_2} S^{(1)}_{I_B I_C s_a l_1 j_1} S^{(2)}_{I_A I_C s_b l_2 j_2} \\
&\times (-)^{I_A - I_C + j_1 - j} \sqrt{(2I_A+1)(2I_B+1)}\; W(I_B j_1 I_A j_2; I_C j) \\
&\times (-)^{s_a + s_b - \sigma - s} W(s_a s_a s_b s_b; \sigma s)(s_a \| \boldsymbol{\sigma}^{(\sigma)}_a \| s_a)(s_b \| \boldsymbol{\sigma}^{(\sigma)}_b \| s_b) \\
&\times (-)^{t_a + t_b - \tau - t} W(t_a t_a t_b t_b; \tau t)(t_a \| \boldsymbol{\tau}^{(\tau)}_a \| t_a)(t_b \| \boldsymbol{\tau}^{(\tau)}_b \| t_b) \\
&\times \frac{2t+1}{2t_a+1} (t_a t \nu_1, \nu_b - \nu_a | t_b \nu_2)(t_a t \nu_a, \nu_b - \nu_a | t_b \nu_b) \\
&\times i^{l - l_1 - l_2} \sqrt{\frac{(2j_1+1)(2j_2+1)(2l+1)(2s+1)(2j+1)}{2I_B+1}} \left\{ \begin{array}{ccc} l_1 & s_a & j_1 \\ l_2 & s_b & j_2 \\ l & s & j \end{array} \right\} \\
&\times (-)^{s_b - m_b} (I_A j M_A, M_B - M_A | I_B M_B)(s_a s_b m_a, -m_b | s m_a - m_b) \\
&\times (lsm, m_a - m_b | j M_B - M_A)\; i^{-l}\; f^{l_1 j_1 l_2 j_2}_{lm}(\mathbf{r}_1, \mathbf{r}_2),
\end{aligned}
$$

where

$$
\begin{aligned}
f^{l_1 j_1 l_2 j_2}_{lm}(\mathbf{r}_1, \mathbf{r}_2) \;&=\; \sum_{m_1} (l_1 l_2 m_1, m - m_1 | l m) Y^{m_1 *}_{l_1}(\hat{r}_1) Y^{m - m_1 *}_{l_2}(\hat{r}_2) \\
&\quad \times u_{l_1 j_1}(r_1) V(r_2) u_{l_2 j_2}(r_2).
\end{aligned}
$$

The spectroscopic amplitude, which must be input, is composed of three parts
a_{lsj} = (spectroscopic amplitude of the target system)
× (spectroscopic amplitude of the projectile system)
× (Clebsch-Gordan coefficients about the isospin).

For the target system

$$
\begin{aligned}
&\langle \alpha_B I_B \| u^{(j)}(j_1, j_2) \| \alpha_A I_A \rangle \\
&= \sum \alpha_C I_C (-)^{I_A - I_C + j_1 - j} \sqrt{(2I_A+1)(2I_B+1)}\; W(I_B j_1 I_A j_2; I_C j) \\
&\times (-)^{n_1 + n_2} \sqrt{n_1} \langle n_1 - 1, \alpha_C I_C : j_1 | \} n_1 \alpha_B I_B \rangle \sqrt{n_2} \langle n_2 - 1, \alpha_C I_C : j_2 | \} n_2 \alpha_A I_A \rangle
\end{aligned}
$$

where $u^{(j)}_m$ is the unit tensor operator

$$
u^j_m(j_1, j_2) = (-)^{2j_2} \frac{1}{\sqrt{2j+1}} [a^+_{j_1} \times \widetilde{a}_{j_2}]^{(j)}_m .
$$

Spectroscopic amplitude of the projectile system is

$$
\begin{aligned}
&(-)^{s_a + s_b - \sigma - s} W(s_a s_a s_b s_b; \sigma s)(s_a \| \boldsymbol{\sigma}^{(\sigma)}_a \| s_a)(s_b \| \boldsymbol{\sigma}^{(\sigma)}_b \| s_b) \\
\times \quad &(-)^{t_a + t_b - \tau - t} W(t_a t_a t_b t_b; \tau t)(t_a \| \boldsymbol{\tau}^{(\tau)}_a \| t_a)(t_b \| \boldsymbol{\tau}^{(\tau)}_b \| t_b).
\end{aligned}
$$

Clebsch-Gordan coefficients about isospin are

$$
\frac{2t+1}{2t_a+1} (t_a t \nu_1, \nu_b - \nu_a | t_b \nu_2)(t_a t \nu_a, \nu_b - \nu_a | t_b \nu_b)
$$

The factors s_i and t_i are given

$$s_1 = \frac{m_A m_B}{m_A m_B - m_a m_b}, \qquad t_1 = \frac{m_b m_B}{m_A m_B - m_a m_b},$$

$$s_2 = \frac{m_a m_A}{m_A m_B - m_a m_b}, \qquad t_1 = \frac{m_A m_B}{m_A m_B - m_a m_b},$$

$$\mathbf{r}_3 = \mathbf{r}_1 - \mathbf{r}_2,$$

$$s_3 = \frac{m_A m_B - m_a m_A}{m_A m_B - m_a m_b}, \qquad t_3 = -\frac{m_A m_B - m_b m_B}{m_A m_B - m_a m_b}.$$

Jacobian J is

$$J = \left[\frac{m_A m_B}{m_A m_B - m_a m_b}\right]^3$$

3.3.1 Nucleon exchange reaction

In the case of nucleon exchange process

$$s_a = s_b = t_a = t_b = 1/2,$$

then

$$(\frac{1}{2}\|\boldsymbol{\sigma}^{(\sigma)}\|\frac{1}{2}) = \sqrt{2(2\sigma+1)}\ , \qquad (\frac{1}{2}\|\boldsymbol{\tau}^{(\tau)}\|\frac{1}{2}) = \sqrt{2(2\tau+1)}\ .$$

Spectroscopic amplitude of projectile

$$\begin{aligned} & (-)^{s_a+s_b-\sigma-s} W(s_a s_a s_b s_b; \sigma s)(s_a\|\boldsymbol{\sigma}_a^{(\sigma)}\|s_a)(s_b\|\boldsymbol{\sigma}_b^{(\sigma)}\|s_b) \\ \times\ & (-)^{t_a+t_b-\tau-t} W(t_a t_a t_b t_b; \tau t)(t_a\|\boldsymbol{\tau}_a^{(\tau)}\|t_a)(t_b\|\boldsymbol{\tau}_b^{(\tau)}\|t_b) \end{aligned}$$

transferred	(σ,τ)			
s and t	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
$(0,0)$	1	3	3	9
$(1,0)$	1	-1	3	-3
$(0,1)$	1	3	-1	-3
$(1,1)$	1	-1	-1	1

Isospin Clebsch-Gordan coefficients

$$\frac{2t+1}{2t_a+1}(t_a t\nu_1, \nu_b - \nu_a | t_b \nu_2)(t_a t\nu_a, \nu_b - \nu_a | t_b \nu_b)$$

	$t=0$	$t=1$	
reaction		neutron excitation	proton excitation
(p,p′)	1/2	−1/2	1/2
(n,n′)	1/2	1/2	−1/2
(p,n)	0	1	(p n^{-1}) excitation
(n,p)	0	1	(n p^{-1}) excitation

ACKNOWLEDGEMENTS

The author wishes his hearty thanks to Professor H. Yoshida for the use of the subroutines of his program DWBA4 [2].

REFERENCES

[1] N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Physical Review **133B** (1964) 3

[2] H. Yoshida, computer code DWBA4, unpublished.

Sample Input

This test data consists of following six examples.

(1) logical test (output 15 pages)

(2) $^{208}\mathrm{Pb}(\mathrm{p},\mathrm{p}')$

- (2-1) macroscopic form factor, where inelastic scattering is induced by the first derivative of the optical potentials (output 4 pages)
- (2-2) microscopic form facter, where projectile proton interacts with the nucleon in the nucleus via Yukawa interaction. Configuration mixing is exemplified. (output 5 pages)
- (2-3) microscopic form factor, with tensor interaction. (output 5 pages)
- (2-4) proton inelastic scattering via (p,d)(d,p') processes. zero range version. (output 6+6+6 pages)
- (2-5) superpose some scattering amplitudes (output 2 pages)

(3) $^{208}\mathrm{Pb}(\mathrm{p},\mathrm{d})$

- (3-1) zero-range one-step calculation (output 4 pages)
- (3-2) finite-range one-step calculation (S-state contribution) (output 5 pages)
- (3-3) finite-range one-step calculation (D-state contribution) (output 5 pages)
- (3-4) finite-range one-step calculation (D-state contribution) (output 4+4 pages)
- (3-5) superpose S-and D-state contributions. D-state contribution alone is also calculated. (output 2+2 pages)

(4) $^{208}\mathrm{Pb}(\mathrm{d},\mathrm{t})$

- (4-1) finite-range form factor (S-state contribution) (output 4 pages)
- (4-2) finite-range form factor (D-state contribution) (output 4 pages)
- (4-3) superpose S-and D-state contributions. (output 2 pages)

(5) $^{208}\mathrm{Pb}(\mathrm{p},\mathrm{t})$

- (5-1) zero-range (p,t) reaction with configuration mixing (output 5 pages)
- (5-2) zero-range (p,d)(d,t) two-step calculation with configuration mixing (output 6+6+2 pages)
- (5-3) finite-range (p,d)(d,t) two-step calculation (output 7 pages)

(6) $^{48}\mathrm{Ca}(^{3}\mathrm{He},\mathrm{t})^{48}\mathrm{Sc}$ (output 10 pages)

Examples (3) to (5) are taken from the Ph.D work of late Dr. Igarashi. Example (6) is recorded here to memorize the dawn of "two-step" work initiated by Dr. M. Toyama.

All the examples are tested on FACOM M780/20 computer with FORTRAN77 compiler. Single precision option is used. Double precision option is advised if one is to use finite range form factor with large orbital angular momentum transfer l.

Double precision option is realized by the following two procedures.
1) Insert double precision statement in each of the subprogram.

```
IMPLICIT DOUBLE PRECISION (A-H,O-Z)   or

IMPLICIT REAL*8 (A-H,O-Z)
```

2) Replace OPEN statements for logical unit numbers 2 and 3 in the main program. old lines

```
OPEN (2,ACCESS='DIRECT',RECL=10792)
OPEN (3,ACCESS='DIRECT',RECL=10792)
```

new lines

```
OPEN (2,ACCESS='DIRECT',RECL=21584)
OPEN (3,ACCESS='DIRECT',RECL=21584)
```

These files are to store finite range form factor table ZFF(152,71).
Input `FORMAT` was modified a little in running these test data to make TSS operation easier. They are, 1) INPUTA subroutine old line

```
2000 READ(5,3000) (A(I),I=1,8)
```

new lines

```
2000 DO 2001 I=1,8
2001 A(I)=0.0
     READ(5,*) A
```

2) INPUTB subroutine old line

```
3000 READ(5,3050,ERR=8000)   A
```

new lines

```
3000 DO 3001 I=1,8
3001 A(I)=0.0
     RED(5,*,ERR=8000) A
```

3) INPUTC subroutine old line

```
 500 READ(5,105) FPUT
```

new lines

```
 500 DO 501 I=1,8
 501 FPUT(I)=0.0
     READ(5,*) FPUT
```

and
old lines

```
     READ(5,110) FMAT
     READ(5,FMAT)  (FFR(I,J),I=1,NRMAXB)
```

new line

```
     READ(5,*)  (FFR(I,J),I=1,NRMAXB)
```

All the blank lines and comment lines which begin with letter 'C' should be deleted before use. The data '*.*' means that the data is not used for this special option.
You can replace *.* by any reasonable number.
Slash '/' tells end of a line. System 2 function tables for (d,p) and (d,t) reactions are supplied in separate file. They include system 2 wave functions for non-orthogonality term analysis.

1) logical test
This is the first data to be run on TWOSTP or TWOFNR. It consists of two types of calculations. The first is the zero-range one step (p,d) reaction, while the second one superposes one- and two-step calculations to get the same result as the first one.
Here are some explanations about this test input.

I-1 title card specifies to output overlap integrals and scattering amplitudes. Rest of the contents is used for users convinience.

I-1.0 Minimum of NRMIN is NOT 0.0. Zero-range option is used.

I-2.2 2.1 card is not necessary.

I-3.1 No reference is made of BETA(1). It is advised to confirm the value of LMAX by inspecting the convergence of overlap integral or cross section.

I-4.1 Q-value is usally 0.0 for this channel.

I-9.0 External input of angle table.

I-10.51 External input of form factor. Specify 10.52 if imaginary part is to be input.

II-1.0 NUBCHN=1 specifies two-step calculation.

II-3.3 BETA(3)=sqrt(4.0*3.14159265). Remember BETA(2)=1.0 is still valid.

II-5.1 Notice VD(1) is 0.0 ! This is the way to replace the old value. VD(1) is defined as a form factor of the first step of the two-step calculation.

II-10.0 You have to put three form factor data. The first is the one for one-step calculation. Second and third form factors are used in two-step calculation. Each form factor set is terminated by a blank card.

```
11                        TWO STEP TEST Pb(p,d)209Pb(1/2 +) One step
1.0   0.0  15.924    1.0  80.0    20.6                      /
2.2   0.5     0.0    0.5                                    /
3.1   0.0    21.0    *.*                                    /
3.2   0.0    21.0    1.0                                    /
4.1   1.0   210.0    1.0   82.0    0.5    0.0    0.0       /
4.2   2.0   209.0    1.0   82.0    1.0    0.5   -4.99      /
5.1  55.761 11.111   6.2   0.0     1.17   0.75   1.188     /
5.2  108.32  20.39   6.2   0.0     1.17   0.75   1.188     /
6.1   1.01    0.75                                         /
6.2   1.01    0.75                                         /
7.1  0.8351   1.32   0.6633                                /
7.2  0.9922   1.29   0.6204                                /
9.0   36.0                                                 /
 0.0   2.5    5.0    7.5   10.0    12.5    15.0    17.5
20.0  22.5   25.0   27.5   30.0    32.5    35.0    37.5
40.0  42.5   45.0   47.5   50.0    52.5    55.0    57.5
60.0  62.5   65.0   67.5   70.0    75.0    80.0    85.0
90.0  95.0  100.0  105.0
0.0    /
```

C a blank or 0.0 card is a terminator for INPUTA

```
10.0   1.0    0.0    0.0   1.0                                  /
10.51  /
 0.31821154E-1 0.29723987E-1 0.27463529E-1 0.24163958E-1 0.20426914E-1
 0.16290836E-1 0.11956777E-1 0.77774543E-2 0.37509301E-2 0.29648194E-3
 -.26918431E-2 -.48860898E-2 -.64322604E-2 -.71781641E-2 -.72665354E-2
 -.67369565E-2 -.57014283E-2 -.43495558E-2 -.27491353E-2 -.11597956E-2
```

```
 0.39596471E-3 0.16793404E-2 0.27044411E-2 0.33257576E-2 0.35793129E-2
 0.34452779E-2 0.29652244E-2 0.22302773E-2 0.12737862E-2 0.24574244E-3
 -.83645619E-3 -.18296340E-2 -.27325950E-2 -.34581022E-2 -.40160362E-2
 -.43868786E-2 -.45852726E-2 -.46381718E-2 -.45600645E-2 -.43939836E-2
 -.41511338E-2 -.38704053E-2 -.35585049E-2 -.32429295E-2 -.29263240E-2
 -.26250365E-2 -.23388844E-2 -.20759106E-2 -.18343751E-2 -.16170947E-2
 -.14217070E-2 -.12482374E-2 -.10942798E-2 -.95865852E-3 -.83921687E-3
 -.73445973E-3 -.64258697E-3 -.56218822E-3 -.49182342E-3 -.43030549E-3
 -.37651137E-3 -.32949349E-3 -.28838753E-3 -.25245606E-3 -.22103911E-3
 -.19356931E-3 -.16954434E-3 -.14853021E-3 -.13014508E-3 -.11405753E-3
 -.99976998E-4 -.87651089E-4 -.76858618E-4 -.67407134E-4 -.59128186E-4
 -.51875002E-4 -.45519148E-4 -.39948594E-4 -.35065365E-4 -.30783798E-4
0.0     /
0                    TWO STEP TEST (Two step)
1.0   1.0  15.924   1.0    80.0   20.6                  /
2.3   0.0     0.0   0.0    0.5    0.0    0.5    0.5     /
3.3   0.0    21.0   3.54491                             /
4.3   1.0   210.0   1.0    82.0   0.5    0.0    0.0     /
5.1   0.0   11.111  6.2    0.0    1.17   0.75   1.188   /
5.3  55.761 11.111  6.2    0.0    1.17   0.75   1.188   /
6.3   1.01   0.75                                       /
7.3  0.8351  1.32  0.6633                              /
0.0                                                     /
```

C the first form factor is the same as that of the one-step

```
10.0   1.0    0.0    0.0   1.0                         /
```

C You may skip the following function table,
C but not the 0.0 card
C this is because the function table is kept in the program

```
10.51  /
 0.31821154E-1 0.29723987E-1 0.27463529E-1 0.24163958E-1 0.20426914E-1
 0.16290836E-1 0.11956777E-1 0.77774543E-2 0.37509301E-2 0.29648194E-3
 -.26918431E-2 -.48860898E-2 -.64322604E-2 -.71781641E-2 -.72665354E-2
 -.67369565E-2 -.57014283E-2 -.43495558E-2 -.27491353E-2 -.11597956E-2
 0.39596471E-3 0.16793404E-2 0.27044411E-2 0.33257576E-2 0.35793129E-2
 0.34452779E-2 0.29652244E-2 0.22302773E-2 0.12737862E-2 0.24574244E-3
 -.83645619E-3 -.18296340E-2 -.27325950E-2 -.34581022E-2 -.40160362E-2
```

```
 -.43868786E-2 -.45852726E-2 -.46381718E-2 -.45600645E-2 -.43939836E-2
 -.41511338E-2 -.38704053E-2 -.35585049E-2 -.32429295E-2 -.29263240E-2
 -.26250365E-2 -.23388844E-2 -.20759106E-2 -.18343751E-2 -.16170947E-2
 -.14217070E-2 -.12482374E-2 -.10942798E-2 -.95865852E-3 -.83921687E-3
 -.73445973E-3 -.64258697E-3 -.56218822E-3 -.49182342E-3 -.43030549E-3
 -.37651137E-3 -.32949349E-3 -.28838753E-3 -.25245606E-3 -.22103911E-3
 -.19356931E-3 -.16954434E-3 -.14853021E-3 -.13014508E-3 -.11405753E-3
 -.99976998E-4 -.87651089E-4 -.76858618E-4 -.67407134E-4 -.59128186E-4
 -.51875002E-4 -.45519148E-4 -.39948594E-4 -.35065365E-4 -.30783798E-4
0.0                                 /
```

C next 4 lines define the first step form factor of the 2-step process

```
10.0   1.0    0.0   1.0    1.0   /
10.11  9.0  55.761  1.17   0.75  /
10.12  0.0   0.0    1.17   0.75  /
0.0                               /
```

C last one for the second step of the two-step process

```
10.0   1.0    0.0   0.0    1.0   /
10.51                             /
 0.31821154E-1 0.29723987E-1 0.27463529E-1 0.24163958E-1 0.20426914E-1
 0.16290836E-1 0.11956777E-1 0.77774543E-2 0.37509301E-2 0.29648194E-3
 -.26918431E-2 -.48860898E-2 -.64322604E-2 -.71781641E-2 -.72665354E-2
 -.67369565E-2 -.57014283E-2 -.43495558E-2 -.27491353E-2 -.11597956E-2
 0.39596471E-3 0.16793404E-2 0.27044411E-2 0.33257576E-2 0.35793129E-2
 0.34452779E-2 0.29652244E-2 0.22302773E-2 0.12737862E-2 0.24574244E-3
 -.83645619E-3 -.18296340E-2 -.27325950E-2 -.34581022E-2 -.40160362E-2
 -.43868786E-2 -.45852726E-2 -.46381718E-2 -.45600645E-2 -.43939836E-2
 -.41511338E-2 -.38704053E-2 -.35585049E-2 -.32429295E-2 -.29263240E-2
 -.26250365E-2 -.23388844E-2 -.20759106E-2 -.18343751E-2 -.16170947E-2
 -.14217070E-2 -.12482374E-2 -.10942798E-2 -.95865852E-3 -.83921687E-3
 -.73445973E-3 -.64258697E-3 -.56218822E-3 -.49182342E-3 -.43030549E-3
 -.37651137E-3 -.32949349E-3 -.28838753E-3 -.25245606E-3 -.22103911E-3
 -.19356931E-3 -.16954434E-3 -.14853021E-3 -.13014508E-3 -.11405753E-3
 -.99976998E-4 -.87651089E-4 -.76858618E-4 -.67407134E-4 -.59128186E-4
 -.51875002E-4 -.45519148E-4 -.39948594E-4 -.35065365E-4 -.30783798E-4
0.0     / end of test data. Good luck !
```

(2) ^{208}Pb(p, p′)
(2-1)macroscopic form-factor
macroscopic form factor, i.e., first derivative of the optical potential

```
0                    208Pb(p,p')208Pb(3- 2.61MeV) Ep=22MeV zero-range
1.0    0.0   20.0    1.0   150.0    22.0   /
2.2    0.0    3.0    3.0                   /
3.1    0.0   20.0    *.*                   /
3.2    0.0   20.0    1.0                   /
4.1    1.0   208.0   1.0    82.0    0.5    /
4.2    1.0   208.0   1.0    82.0    0.5    3.0    -2.61    /
5.1   57.6   10.98   6.2     0.0    1.17   0.75    1.25    /
5.2   57.6   10.98   6.2     0.0    1.17   0.75    1.25    /
6.1    1.0    0.75                         /
6.2    1.0    0.75                         /
7.1   0.80    1.32   0.66                  /
7.2   0.80    1.32   0.66                  /
9.0   36.0    5.0    0.0                   /
0.0                                        /
10.0   0.2    0.0    1.0     1.0           / beta3 = 0.2 here
```

C first derivatives of the optical potentials for form factor

```
10.11  3.0   57.6    1.17   0.75          /
10.12  3.0   10.98   1.32   0.66   *.*    *.*    0.805    /
0.0                                        /
```

(2-2) microscopic form factor with Yukawa interaction
C nuclear structure(one-particle one-hole state)
C $3^-\rangle = 0.47 \mid \pi 0h9/2\ \pi^{-1}1d3/2\rangle - 0.35 \mid \pi 1f7/2\ \pi^{-1}2s1/2\rangle$
C $\quad +0.42 \mid \nu 1g9/2\ \nu^{-1}2p3/2\rangle - 0.39 \mid \nu 0i13/2\ \nu^{-1}1f5/2\rangle$
C ref. V. Gillet Nucl. Phys. 88(1966) 321
C these amplitudes are superposed
C Yukawa interaction with strength = −50MeV, range= 1fm^{-1}
C scattering amplitude is to be output onto permanent file
C optical potential etc. are the same as above and are neglected
C

```
5          1        208Pb(p,p') Ep=22MeV, 3- at 2.61 MeV
0.0  /  all the INPUTA data are the same as the former, but the title line
```

```
10.0    0.47   1.0   *.*    0.0    0.0    22.0    22.0    /
10.61   1.0   50.0   1.0                                   /
10.62   1.0   207.0                                        /
10.63   3.77   0.0   5.0    4.5    81.0                    / π0h9/2
10.64   1.17   1.25  0.75   6.0                            /
10.65   8.38   1.0   2.0    1.5    81.0                    / π1d3/2
10.66   1.17   1.25  0.75   6.0                            /
0.0                                                       /
10.0   -0.35   1.0   *.*    0.0    0.0    22.0    22.0    /
10.63   2.89   1.0   3.0    3.5    81.0                    / π1f7/2
10.65   8.03   2.0   0.0    0.5    81.0                    / π2s1/2
0.0                                                       /
10.0    0.42   1.0   *.*    0.0    0.0    22.0    22.0    /
10.63   3.94   1.0   4.0    4.5    0.0                     / ν1g9/2
10.65   8.27   2.0   1.0    1.5    0.0                     / ν2p1/2
0.0                                                       /
10.0   -0.39   0.0   *.*    0.0    0.0    22.0    22.0    /
10.63   3.15   0.0   6.0    6.5    0.0                     / ν0i13/2
10.65   7.95   1.0   3.0    2.5    0.0                     / ν1f5/2
0.0                                                       /
```

(2-3) microscopic form factor with tensor interaction
C
C OPEP tensor interaction with strength= -30MeV and force range = 0.7fm^{-1}
C transferred s in line 2.2 is changed from 0 to 1
C rest of the BASIC data are exactly the same as above
C interaction type is changed from Yukawa to OPEP tensor
C nuclear structure is the same as above
C cards 10.62, 10.64 and 10.66 need not be input again
C this scattering amp. is NOT output onto permanent file
C

```
0                  208Pb(p,p) 3- at 2.61 MeV  Ep=22MeV L=3, S=1
2.2    1.0    3.0    3.0    /
0.0                          /
10.0    0.47    1.0    *.*    0.0    0.0    22.0    22.0    /
10.61   1.0   -30.0    0.7                                   /
10.63   3.77    0.0    5.0    4.5    81.0                    /
10.65   8.38    1.0    2.0    1.5    81.0                    /
0.0                                                         /
10.0   -0.35    1.0    *.*    0.0    0.0    22.0    22.0    /
```

```
10.63   2.89    1.0    3.0    3.5    81.0                    /
10.65   8.03    2.0    0.0    0.5    81.0                    /
0.0                                                           /
10.0    0.42    1.0    *.*    0.0    0.0    22.0    22.0     /
10.63   3.94    1.0    4.0    4.5    0.0                     /
10.65   8.27    2.0    1.0    1.5    0.0                     /
0.0                                                           /
10.0   -0.39    0.0    *.*    0.0    0.0    22.0    22.0     /
10.63   3.15    0.0    6.0    6.5    0.0                     /
10.65   7.95    1.0    3.0    2.5    0.0                     /
0.0                                                           /
```

(2-4) two-step via (p,d)(d,p) channel
C
C zero-range two-step process (p,d)(d,p')
C promoting 1d3/2 proton to 0h9/2
C promoting 2s1/2 proton to 1f7/2
C promoting 2p3/2 neutron to 1g9/2
C promoting 1f5/2 neutron to 0i11/2
C these four scattering amplitudes should be superposed
C QVLUE(3) is assumed to be that of the (p,d) ground state
C

```
5          2        208Pb(p,d)(d,p) 3- at 2.61MeV  Ep=22MeV
1.0    1.0    20.0   1.0   150.0  22.0                    /
2.3    0.5    2.0    1.5    0.5    5.0    4.5     3.0    /
3.2    0.0    20.0   0.0                                  /
3.3    0.0    20.0   1.0                                  /
4.3    2.0    207.0  1.0    82.0   1.0    1.5    -5.15   /
5.3   112.0   19.4   6.0    0.0    1.25   0.682   1.25   /
6.3    1.12   0.47                                       /
7.3    1.0    1.25  0.783                                 /
0.0                                                      /
10.0   1.0    0.0  15300.0  1.0                          /
10.41  1.0   81.0    8.38   1.0   207.0                   /
10.42  1.17   1.2    0.75   6.0                          /
0.0                                                      /
10.41  0.0   81.0    3.77   1.0   207.0                   /
0.0                                                      /
5          3        208Pb(p,d)(d,p) 3- at 2.61MeV  Ep=22MeV
2.3    0.5    0.0    0.5    0.5    3.0    3.5    3.0     /
4.3    2.0   207.0   1.0    82.0   1.0    0.5   -5.15    /
```

```
0.0                                                              /
10.41  2.0    81.0   8.03   1.0   207.0                          /
0.0                                                              /
10.41  1.0    81.0   2.89   1.0   207.0                          /
0.0                                                              /
5          4        208Pb(p,d)(d,p) 3- at 2.61MeV  Ep=22MeV
2.3    0.5    1.0    1.5    0.5    5.0    4.5    3.0     /
4.3    2.0   207.0   1.0   82.0    1.0    1.5   -5.15    /
0.0                                                              /
10.41  2.0    0.0    8.27   1.0   207.0                          /
0.0                                                              /
10.41  1.0    0.0    3.94   1.0   207.0                          /
0.0                                                              /
5          5        208Pb(p,d)(d,p) 3- at 2.61MeV  Ep=22MeV
2.3    0.5    3.0    2.5    0.5    6.0    5.5    3.0     /
4.3    2.0   207.0   1.0   82.0    1.0    2.5   -5.15    /
0.0                                                              /
10.41  1.0    0.0    7.95   1.0   207.0                          /
0.0                                                              /
10.41  0.0    0.0    3.94   1.0   207.0                          /
0.0                                                              /
```

(2-5) superpose scattering amplitudes
C superposes inelastic scattering by Yukawa int. and (p,d)(d,p) two-step
C processes

```
9                   Subroutine mix is in order
1.0    1.0    0.0    /
2.0    0.47   0.0    /
3.0    -0.35  0.0    /
4.0    0.42   0.0    /
5.0    -0.39  0.0    /
0.0                  /
```

(3) ^{208}Pb(p, d)
(3-1) zero-range one-step calculation

```
0                   208Pb(p,d)207Pb(2p1/2 Ex=0.0MeV) Ep=22MeV zero-range
1.0    0.0     20.0    1.0     150.0   22.0      /
2.2    0.5     1.0     0.5                       /
3.1    0.0     20.0                              /
3.2    0.0     20.0    1.0                       /
4.1    1.0     208.0   1.0     82.0    0.5       /
```

```
4.2    2.0     207.0   1.0     82.0    1.0     0.5   -5.15 /
5.1    57.6    10.98   6.2     0.0     1.17    0.75   1.25 /
6.1    1.01    0.75                                  /
7.1    0.805   1.32    0.66                          /
5.2    112.0   19.4    6.0     0.0     1.25    0.682  1.25 /
6.2    1.12    0.47                                  /
7.2    1.0     1.25    0.783                         /
9.0    36.0    5.0     0.0                           /
0.0                                                  /
10.0   1.0     0.0   15300.0   1.0                   /
10.41  2.0     0.0     8.38    1.0     207.0         /
10.42  1.17    1.25    0.75    6.0                   /
0.0                                                  /
```

(3-2) finite-range one-step calculation (S-state contribution)

```
5          1        208Pb(p,d)207Pb(1/2-, gs) finite range S-state
1.0    0.0     20.0    1.0     150.0   22.0    1.0       /
0.0                                                      /
0 0 0 0 0 0 0 0 0 1
1.0    1.0     20.0    1.0     2.0                       /
2.0    2.0     1.0     0.5     0.0     0.0     0.5   0.5 /
3.0    1.0     1.0     8.38    2.22                      /
4.0    2.0                                               /
5.0    1.0     207.0   0.0     1.17    0.75    6.0       /
6.0    1.0     1.0     0.0     1.17    0.75    6.0       /
8.2    0.0375  0.0     / REID SOFT CORE POT. (P,D) S-STATE
```

C you are requested to place Vnp*(deuteron internal wf) here
C full function table can be found in PDRSC.DAT file
C ID=1 is for S-state

```
0.39969160E+2 0.69233626E+2 0.10879197E+3 0.15238540E+3 0.19347038E+3
***   ***   ***   ***   ***   ***   ***   ***   ***   ***   ***
-.70466498E-7 -.67678647E-7 -.65002067E-7 -.62432274E-7 -.59964965E-7
0.0      /
```

(3-3) finite-range one-step calculation (D-state contribution)

```
5          2       208Pb(p,d)207Pb(1/2-, gs) finite range D-state
2.2    1.5     1.0     0.5     /
```

```
0.0                              /
0 0 0 0 0 0 0 0 0 0
2.0    2.0     1.0     0.5     0.0     2.0     1.5    0.5 /
8.2    0.0375  0.0     /D-state\\
```

C you are requested to place Vnp*(deuteron internal wf) here
C full function table can be found in PDRSC.DAT file
C ID=2 is for D-state

```
0.17046324E+1 0.10390344E+2 0.20753832E+2 0.31828517E+2 0.41662972E+2
***   ***   ***   ***   ***   ***   ***   ***   ***   ***   ***
-.21200549E-6 -.20357311E-6 -.19547916E-6 -.18770991E-6 -.18025221E-6
0.0      /
```

(3-4) finite-range one-step calculation (D-state contribution)

```
5          3       208Pb(p,d)207Pb(1/2-, gs) finite range D-state
2.2      1.5       2.0       0.5       /
0.0                                    /
0 0 0 0 0 0 0 0 0 0
8.2      0.0375    1.0       /D-state. Use old wf
0.0      /
```

(3-5) superpose S- and D-state contributions

```
9                   MIX is to be called
1.0   1.0    0.0     /
2.0   1.0    0.0     /
3.0   1.0    0.0     /
0.0   /
```

C next 3 lines are to calculate D-state contribution

```
2.0   1.0    0.0     /
3.0   1.0    0.0     /
0.0   /
```

(4) ^{208}Pb(d, t)
4-1) finite-range form factor (S-state contribution)

```
5            1141980 208PB(D,T)207PB(1/2- G.S.) ED=17MEV S-STATE RSC5
1.0   0.0   20.0   1.0  100.0   17.0   1.0   /
2.2   0.5   1.0    0.5                       /
3.1   0.0   22.0                             /
3.2   0.0   26.0   1.0                       /
4.1   2.0   208.0  1.0   82.0   1.0    0.0          /
4.2   3.0   207.0  1.0   82.0   0.5    0.5   -1.11  /
5.1 109.9   9.8    5.25  0.0   1.063  1.038   1.3   /
5.2  161.7  19.6   2.0   0.0    1.2    0.72   1.3   /
6.1   0.9   0.6              /
6.2   1.2   0.72             /
7.1   1.0  1.501   0.728      /
7.2   0.0   1.4    0.86       /
9.0  36.0   5.0    0.0        /
0                             /
0
1.0   1.0  20.0   2.0   1.0   /
2.0   2.0   1.0   0.5   0.0   0.0   0.5    0.5  /
3.0   1.0   1.0 7.3677  6.257                   /
4.0   2.0                                       /
5.0   1.0 207.0   0.0  1.225  0.7   6.5         /
6.0   1.0   2.0   0.0   2.0   0.65  0.0         /
8.2   0.03  0.0   / RSC 5 (d,t) S-state
```

C place 400(=NRMAXB) data table for (d,t) reactions
C you can find full table for the case of Reid soft core potential
C in separate file DTRSC5.DAT

```
-6.79447376D+0 -6.88729498D+0 -6.96984828D+0 -7.04212786D+0 -7.12357354D+0
***     ***     ***     ***     ***     ***     ***     ***     ***
-1.88865988D-7 -1.81311699D-7 -1.74886680D-7 -1.65641569D-7 -1.57245474D-7
/
```

(4-2) finite-range form factor (D-state contribution)

```
5            1141980 208PB(D,T)207PB(1/2- G.S.) ED=17MEV D-STATE  L=1
2.2   1.5  1.0  0.5  /
0.0                 /
0
1.0   1.0  20.0   2.0   1.0     /
2.0   2.0   1.0   0.5   0.0   2.0   1.5    0.5  /
8.2   0.03  0.0   /   L=2 part of RSC5 (d,t) system 2 function
-1.06744234D-2 -2.57846447D-2 -5.11596044D-2 -9.16680225D-2 -1.32438181D-1
```

```
***     ***     ***     ***     ***     ***     ***     ***     ***
-4.95080138D-7 -4.68177555D-7 -4.34699629D-7 -4.20774835D-7 -3.99804185D-7
/
```

(4-3) superpose S- and D-state contributions

```
9                   Mix is in order
1.0   1.0   0.0   /
2.0   1.0   0.0   /
0     /
```

(5) ^{208}Pb(p, t)
(5-1) zero-range (p,t) reaction with configuration mixing

```
5                   208Pb(p,t)206Pb(2+ 0.8MeV) 0.724(p1/2f5/2) -.523(p1/2p3/2)
1.0     0.0     20.0    1.0     150.0   22.0    /
2.2     0.0     2.0     2.0                     /
3.1     0.0     20.0                            /
3.2     0.0     20.0    1.0                     /
4.1     1.0     208.0   1.0     82.0    0.5     /
4.2     3.0     206.0   1.0     82.0    0.5     2.0    -6.426   /
5.1     57.6    10.98   6.2     0.0     1.17    0.75    1.25    /
5.2     160.9   17.3    2.5     0.0     1.2     0.72    1.3     /
6.1     1.01    0.75                    /
6.2     1.2     0.72                    /
7.1     0.805   1.32    0.66            /
7.2     0.0     1.40    0.84            /
9.0     37.0    5.0     0.0             /
0.0                                     /
10.0    0.724   1.0     1.0     1.0     /
10.71   0.243                           /
10.72   1.0     206.0                   /
10.73   7.4539  2.0     1.0     0.5     /
10.74   1.25    1.25    0.65    6.0     /
10.75   7.4539  1.0     3.0     2.5     /
10.76   1.25    1.25    0.65    6.0     /
0.0                                     /
10.0    -0.523  0.0     1.0     1.0     /
10.75   7.4539  2.0     1.0     1.5     /
0.0     /
```

(5-2) zero-range (p,d)(d,t) two-step calculation with configuration mixing

```
5                  208Pb(p,d)207Pb(1/2-)(d,t)206Pb(2+ 0.8MeV) Ep=22MeV
1.0     1.0      20.0     1.0      150.0   22.0     /
2.3     0.5      1.0      0.5      0.5     3.0      2.5   2.0    /
3.2     0.0      20.0     0.0      /
3.3     0.0      20.0     1.0      /
4.3     2.0      207.0    1.0      82.0    1.0      0.5   -5.145 /
5.3     105.8    19.68    0.0      0.0     1.15     0.81   1.15  /
7.3     0.0      1.34     0.68     /
0.0                               /
10.0    1.0      0.0      15300.0  1.0     /
10.41   2.0      0.0      7.4539   1.0     206.0    /
10.42   1.25     1.25     0.65     6.0     /
0.0                                       /
10.0    0.724    0.0      5.06E+4  1.0     /
10.41   1.0      0.0      7.4539   1.0     206.0    /
0.0     /
5                  208Pb(p,d)207Pb(1/2-)(d,t)206Pb(2+ .8MeV) Ep=22MeV
2.3     0.5      1.0    0.5       0.5    1.0     1.5   2.0 /
0.0                               /
10.0    1.0      0.0    1.53E+4   1.0    /
10.41   2.0      0.0    7.4539    1.0    206.0   /
0.0                                     /
10.0    -0.523   0.0    5.06E+4   1.0    /
10.41   2.0      0.0    7.4539    1.0    206.0   /
0.0                                     /
9                  mix is in order
2.0   1.0   /
3.0   1.0   /
0           /
-9          / exit from MIX and read new title data
```

(5-3) finite-range (p,d)(d,t) two-step calculation

```
5        1      208Pb(p,d)(d,t)206Pb(3- 1.34MeV) Ep=22MeV
1.0    1.0     20.0    1.0     150.0  22.0   1.0   1.0 /
2.3    0.5     1.0     0.5     0.5    3.0    3.5   3.0 /
3.1    0.0     20.0    0.0     /
3.2    0.0     20.0    0.0     /
3.3    0.0     20.0    1.0     /
4.1    1.0     208.0   1.0     82.0   0.5    0.0   0.0 /
4.2    3.0     206.0   1.0     82.0   0.5    3.0  -4.29 /
4.3    2.0     207.0   1.0     82.0   1.0    0.5  -5.15 /
5.1    51.8    10.0    6.0     0.0    1.25   0.65  1.25 /
```

```
6.1    1.12    0.47           /
7.1    1.0     1.25    0.76   /
5.2    168.9   9.9     6.0    0.0    1.20   0.65  1.30 /
6.2    1.15    0.92           /
7.2    0.0     1.60    0.97   /
5.3    112.0   19.4    6.0    0.0    1.25  0.682  1.25 /
6.3    1.12    0.47           /
7.3    1.0     1.25    0.783  /
9.0    37.0    5.0     0.0    /
0.0                           /
0 0 0 0 0 0 0 0 0 1
1.0    1.0    20.0    1.0    1.0    /
2.0    2.0    1.0     0.5    0.0    0.0    0.5   0.5 /
3.0    1.0    1.0     5.18   2.22   /
4.0    2.0                          /
5.0    1.0    206.0   0.0    1.25   0.65   6.0 /
6.0    1.0    1.0     0.0    1.25   0.65   6.0 /
8.2  0.0375   0.0   / REID SOFT CORE POT. (P,D) S-STATE
```

C reference should be made of the S-state form factor of (p,d) reaction
C see example (3-2)

```
0.0  /  end of one set of form factor data
0 0 0 0 0 0 0 0 0 1
2.0   1.0    3.0    3.5    0.0    0.0    0.5   0.5 /
8.2   0.03   0.0   / RSC 5 (d,t) S-state
```

C refer to the test input of (d,t) reaction
C see example (4-1)

```
  0.0
```

(6) ^{48}Ca(^{3}He, t)^{48}Sc

```
11                    48Ca(3He-4He-t)48Sc 0+  E=23.0 MeV
1.0     1.0     20.0    1.0     100.0   23.0   /
2.3     0.5     3.0     3.5     0.5     3.0     3.5   /
3.1     0.0     30.0    0.0     /
3.2     0.0     30.0    0.0     /
3.3     0.0     30.0    1.0     /
4.1     3.0     48.0    2.0     20.0    0.5     /
4.2     3.0     48.0    1.0     21.0    0.5     0.0     -6.41  /
4.3     4.0     47.0    2.0     20.0    0.0     3.5     10.64  /
5.1     152.3   22.3    0.0     0.0     1.22    0.695   1.3    /
```

```
5.2     142.3   24.5    0.0     0.0     1.22    0.695   1.3     /
5.3     183.7   26.0    0.0     0.0     1.4     0.56    1.4     /
7.1     -0.06   1.506   0.8     /
7.2     0.04    1.506   0.8     /
7.3     0.0     1.48    0.56    /
9.0     36.0    2.0     2.0     /
0.0   /
10.0    1.0     0.0     0.0     1.0     /
10.41   0.0     0.0     9.94    1.0     47.0    /
10.42   1.25    1.4     0.65    6.25    /
0.0    /
10.41   0.0     20.0    2.77    1.0     47.0    /
0.0    /
```